表面張力的誤差探討與實驗改進

獲獎榮譽榜

參加美國第五十二屆國際科技展覽會獲大會獎物理科三等獎
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從小就喜歡看科學方面的書籍，就讀於國小國中時，幸蒙老師的鼓勵和父母的支持，一直參加科學展覽，原本只是抱著作幾個實驗玩玩的態度，沒想到開始研究後竟對整個研究的「過程」著了迷，從此便和科展結下了不解之緣。

今年很幸運的能以自己發明的「表面張力測定儀」代表國家參加第五十二屆美國科技展覽會，首先要感謝的是詹國禎教授、戴敏章老師所給予的指導，當然當初一再鼓勵我繼續科展研究的徐正梅導師無疑的是促使這件作品誕生的推手，我的心中除了感謝還是感謝！

在科展的研究中每每會遇到一些無法克服的問題，當苦思無解而有放棄的念頭時，父親總會提醒「即使是覺得沒有希望，還是要拼到最後一秒，不到最後，絕不能輕言放棄」。的確，每當我半信半疑的繼續往下走，總會有令我意想不到的收穫，此次的參展除了再一次堅定了我要做就要「拼到底」的信念，更令我深知書到用時方恨少，所見實在太少，日後我會繼續努力研究的！

研究動機

表面張力的測量在液體和氣體介面間分子作用力的研究佔有重要的地位。由於都諾表面張力測定儀能夠經由簡單的操作而得到精確的結果，所以為目前各個大學廣為使用，但是經由實驗發現此測定儀並不能測量水(標準液體)的表面張力也不能準確的將測量環離開液面的瞬間做精確的測量，此外實驗過程中並沒有考慮到力臂的改變。基於以上的種種理由希望能夠設計出一台能夠自動測試且能將原測量方法更為改進的新型表面張力測定儀。

理論及方法

都諾表面張力測定儀的測量原理

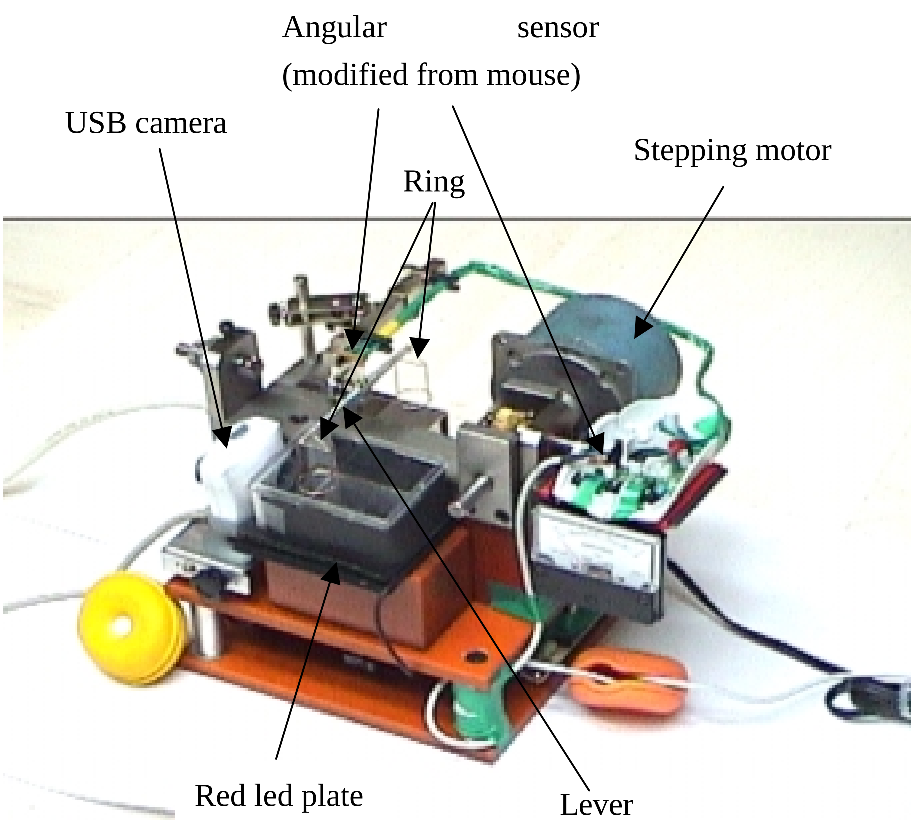
都諾表面張力測定儀的測量方式是使用典型的圓環法作為基礎，在校準後，轉動旋轉軸帶動鋼絲拉動測量環脫離水面同時測得該待測液體的表面張力值，其所使用的原理如下：

$$\gamma = \gamma_w \times \frac{\theta}{\theta_w} \qquad (1)$$

其中γ(γ_w)是表面張力值而θ (θ_w)則是儀器所測得的扭轉角

照片一、都諾表面張力測定儀

新型表面張力測量儀之原理

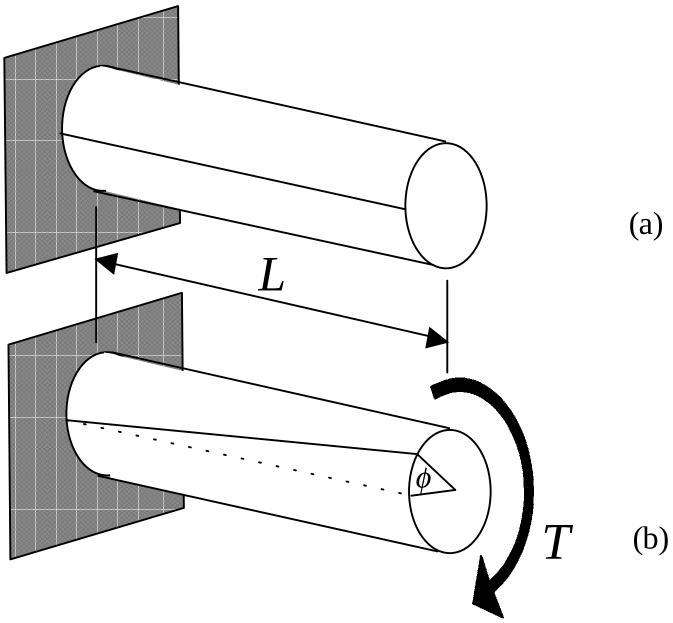


照片二、新型表面張力測定儀

新型表面張力測定儀的測量原理是從彈性力學中推導而來。如圖一(a)所示，若一個扭力矩 T 作用於一端固定的圓柱體，則會產生一個扭轉角∮如圖一(b)，而其關係如下：

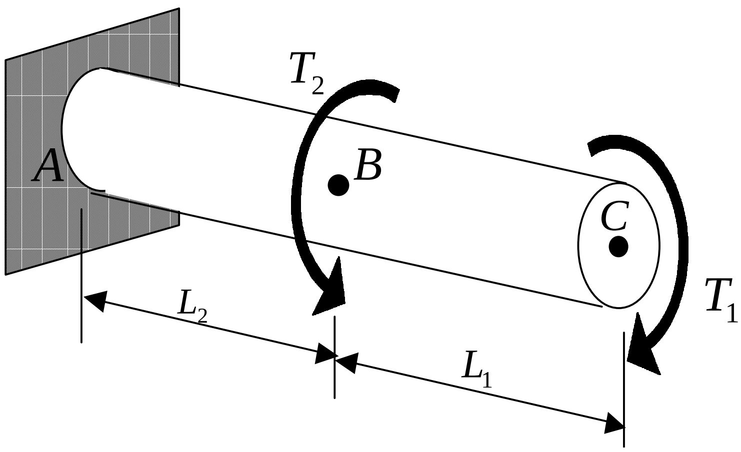
$$\phi = \frac{T \cdot L}{G \cdot J} \qquad (2)$$

其中G為剛性係數、J為面積極慣性距而L為所擷取斷面的長度，而從(2)式可知扭轉角正比於扭力矩，所以扭力矩可以經由測量扭轉角而獲得。



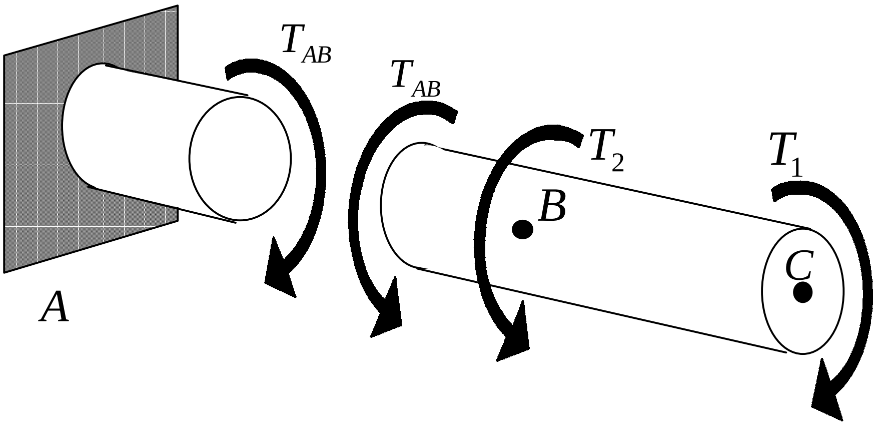
圖一、彈性力學說明圖

圖二為新型表面張力測定儀的模型，其中T_1 為步進馬達傳至扭轉軸的扭力矩、T_2 為表面張力所產生的扭力矩而 ϕ_B 和 ϕ_C 分別為固定端和扭轉端的扭轉角。



圖二、新型表面張力測定儀說明圖

為了更仔細的說明將模型分解為圖三和圖四。



圖三、A、B 間的力圖

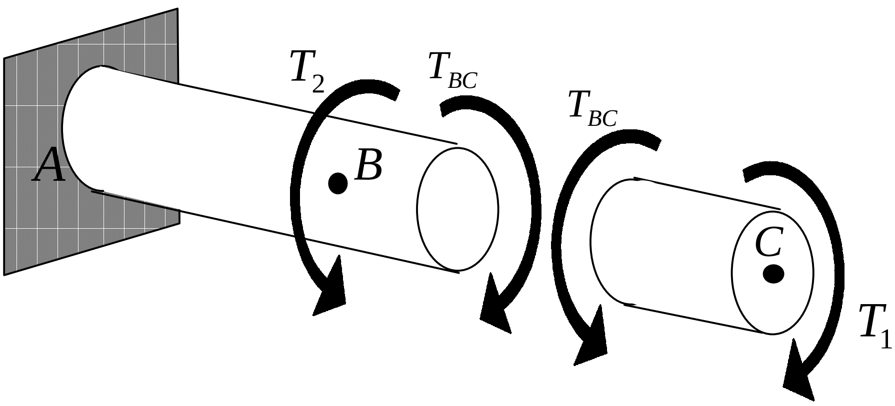
在 A 和 B 之間任擷取一處， T_{AB} 為該處的扭力矩，而藉由扭力矩平衡可以推得下式：

$$T_{AB} + T_2 - T_1 = 0 \tag{3}$$

將(2)式代入(3)

$$\phi_B = \frac{T_{AB}L_2}{GJ} = \frac{(T_1 - T_2)L}{GJ} \tag{4}$$

同樣的在 B、 C 間任取一處我們可以做如下的推導



圖四、B、C 間的力圖

$$T_{BC} - T_1 = 0 \quad (5)$$

$$\phi_{C/B} = \frac{T_{BC} \cdot L_1}{G \cdot J} = \frac{T_1 \cdot L_1}{G \cdot J} \quad (6)$$

接著將相對關係轉換為絕對關係

$$\phi_C = \phi_{C/B} + \phi_B \quad (7)$$

將(4)和(6)代入(7)

$$\phi_C = \frac{(T_1 - T_2) \cdot L_2}{G \cdot J} + \frac{T_1 \cdot L_1}{G \cdot J} \quad (8)$$

解聯立方程組(4)和(8)可得

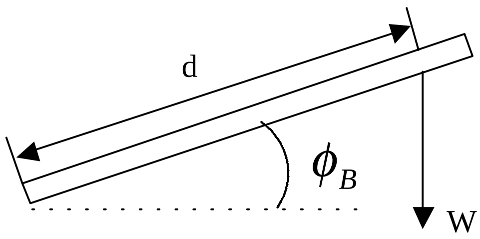
$$T_1 = G \cdot J \cdot \frac{\phi_C - \phi_B}{L_1} \quad (9)$$

$$T_2 = G \cdot J \cdot (\frac{\phi_C - \phi_B}{L_1} - \frac{\phi_B}{L_2}) \quad (10)$$

若 $L_1 = L_2 = L$ 則

$$T_2 = \frac{G \cdot J}{L} \cdot (\phi_c - 2 \cdot \phi_B) = K \cdot (\phi_c - 2 \cdot \phi_B) \quad (11)$$

測量 ϕ_B 和 ϕ_C 即可得到 T_2，而對於力臂變化所造成的誤差則可以修正如下



圖五、力矩的修正

在圖五中 d 為桿長而 W 為表面張力

$$T_2 = W \cdot d \cdot Cos\phi_B \quad (12)$$

$$W = 2\pi \cdot (R_i + R_o) \cdot \gamma \quad (13)$$

解方程組(11)、(12)、(13)表面張力值 γ 即可測得

$$\gamma = \frac{K \cdot (\phi_C - 2 \cdot \phi_B)}{2\pi \cdot (R_i + R_o) \cdot d \cdot Cos\phi_B} = K_1 \cdot \frac{\phi_C - 2 \cdot \phi_B}{Cos\phi_B} \quad (14)$$

實驗器材

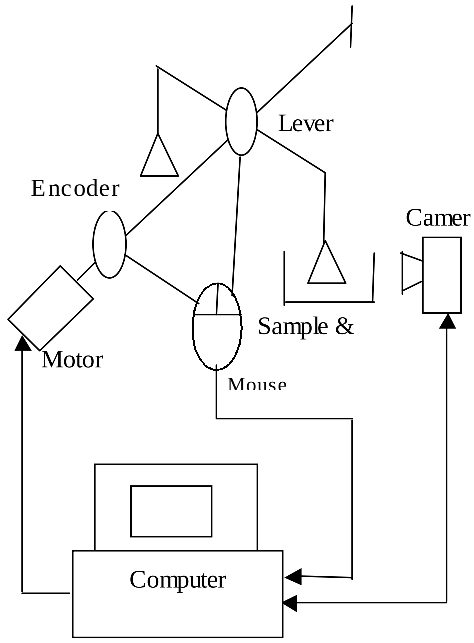
1.實驗環和容器(裝待測液體用)
2.步進馬達及其電源供應器
3.鋼絲(數條)
4.筆記型電腦(有 LPT 埠)
5.I-com (USB 介面的數位像機)

圖六為整個裝置的關係圖，其主要的改進部分詳述如下：

1.利用新型的表面張力測定儀可以自動化的進行測量的工作，經由 LTP 埠的聯繫，扭轉軸由電腦控制的步進馬達進行持續且穩定的轉動，不僅排除了手動操作的不穩定性，且達到了省時的目的。

2.由滑鼠 X 軸和 Y 軸感測器改裝而成的角度感測器分別裝製於固定端籍測量桿端，可以準確的測之各個時間桿子上所作用的扭力矩值。

3.由 USB 介面的數位像機抓取影像，再經由特殊的演算方法可以準確的得知測量環離開液面的時刻，且利用影像擷取的技術可以將整個實驗過程做準確的紀錄。



圖六、新型表面張力儀的關係圖

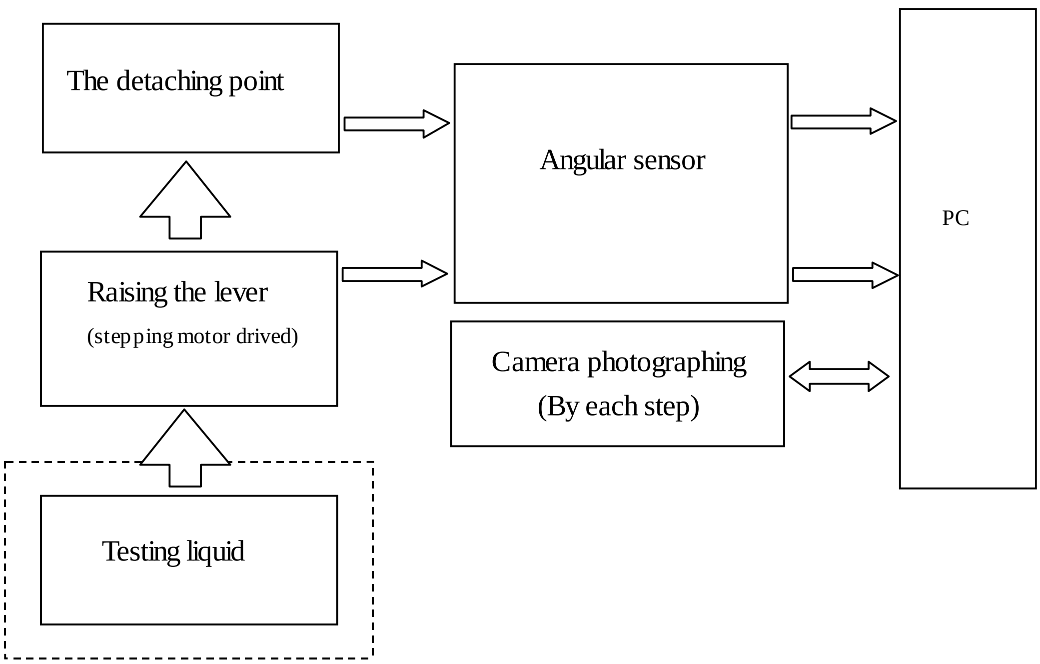
實驗過程與方法

A. 實驗樣品

1. 純水
2. 酒精

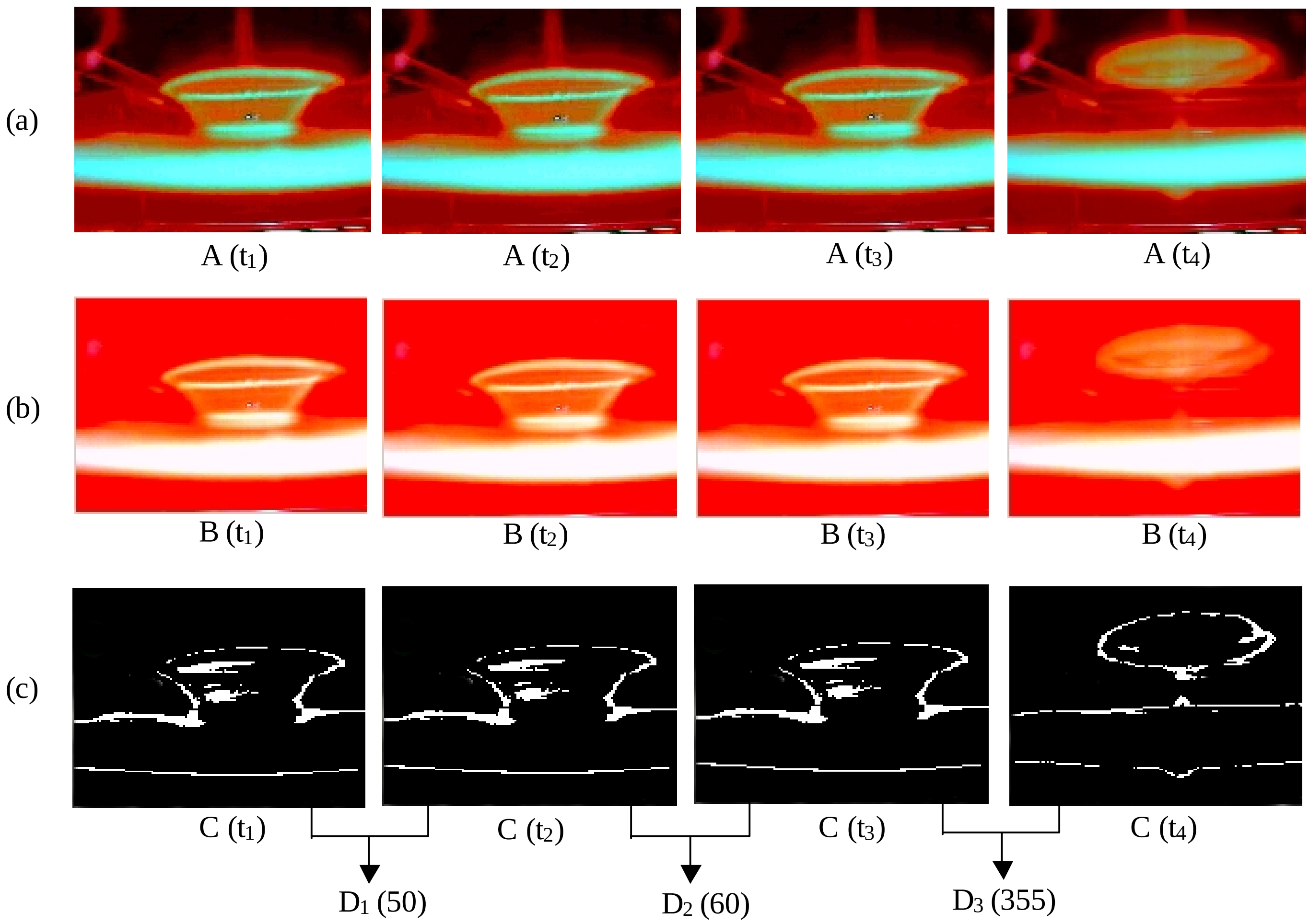
B. 實驗方法

整個實驗的流程圖如圖七所示：



圖七、實驗流程圖

當實驗開始時，測量桿會經由步進馬達的驅動而向上舉起直到測量環脫離液面。在整個實驗過程中，數位像機會和步進馬達同步的記錄影像，同時利用 Visual Basic 所撰寫的程式會對所得到的影像進行分析。其機制是將每一次所抓取到的影像的紅色圖層經由合適的臨界值轉換成二進位影像，接著再比較每連續兩張的數位影像，記錄其中相異的二進位碼的數目作為比較其差異度的基準，若連續兩張二進位影像的差異度突然有巨幅增加則可以斷定測量環離開液面的準確時間(如圖八所示，其中 D 為相異的二進位碼數)。



圖八 (a)由數位像機所抓取的影像 (b)紅色圖層 (c)二進位影像圖

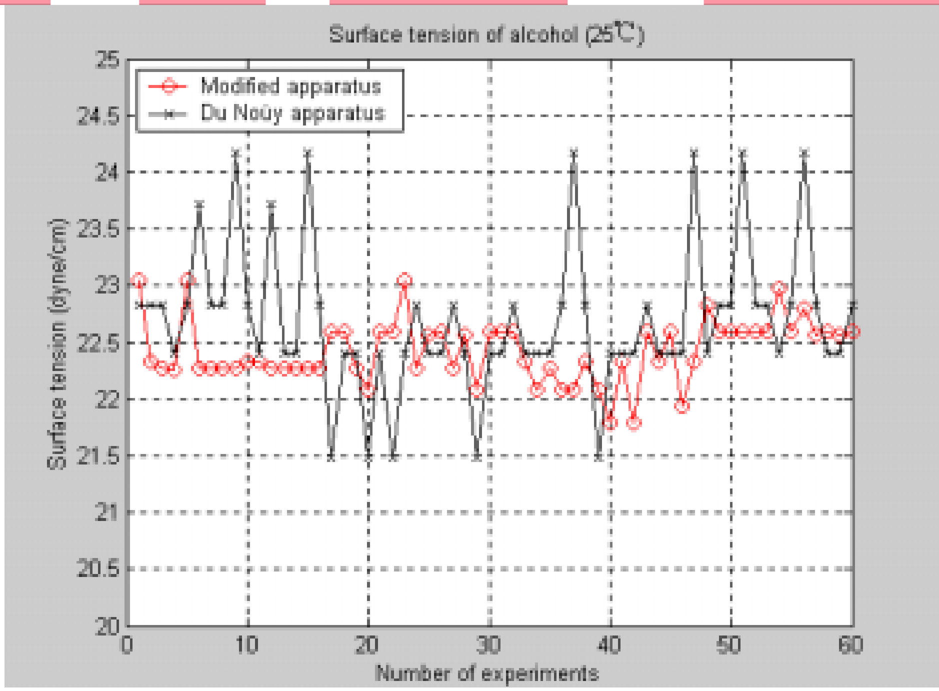
接著再搭配上之前所改裝的角度感測器讀取固定端和測量桿端的扭轉角度即可以直接求得該液體的表面張力值而不須經由基準表的協助。

此外整個實驗的自動化也同時完成，當實驗結束時，所得數據的平均值、標準差和圖形可以馬上呈現，達到了省時的目的。

實驗結果及討論

A.和都諾表面張力測定儀準確度的比較

經由新型的表面張力測定儀亦可以使用原都諾表面張力測定儀的測量方式來實驗，所以在第一個實驗中使用兩種不同的儀器用同一種方式測量酒精的表面張力值，其結果如圖九：



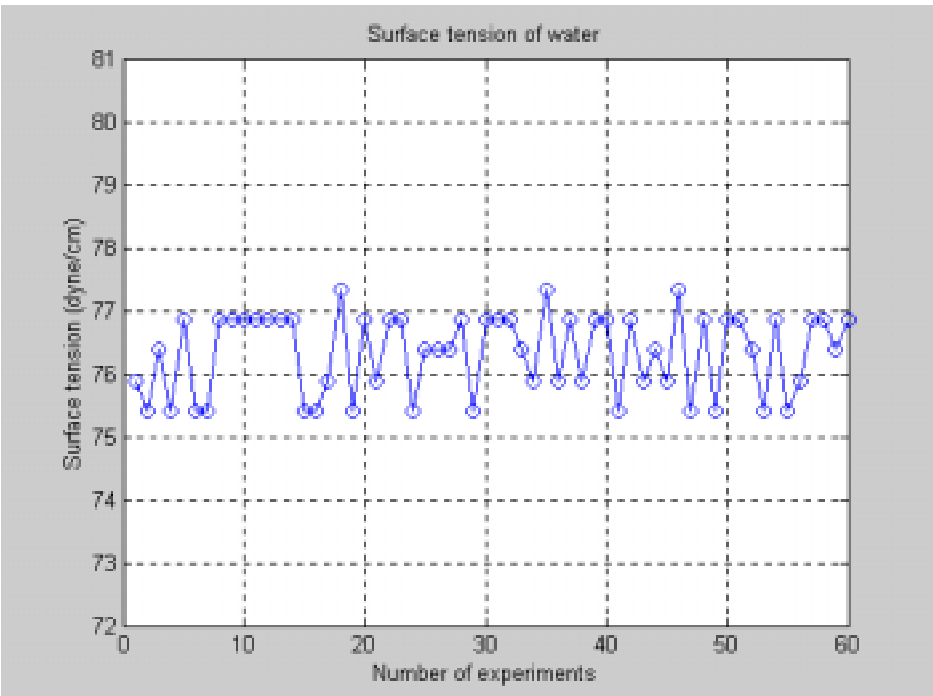
圖九、兩表面張力測量儀的比較(使用相對法)

由60次實驗的折線圖中可以發現新型的表面張力測定器所測得的數據的標準差明顯降低，其原因應由於角度感測器的精準度較原本更為精準且經由步進馬達驅動使整個實驗過程較為穩定。

新型表面張力張力測定儀所得到的實驗數據介於 23.0 和 21.8 其所含括的範圍較都諾表面張力測定儀的 24.2 ~ 21.5 為小，其原因可能為測量環脫離瞬間的準確測量。

新型表力測定儀所測得的表面張力值為 22.42 ± 0.28 (dyne/cm) 較都諾表面張力測定儀所得到的 22.7 ± 0.7 (dyne/cm) 更為接近目前的公認值 22.39 dyne/cm.

B. 利用新型表面張力測定儀測量水(原標準液體)



圖十、利用新型表面張力測定儀測量水的表面張力

如圖十所測得水的表面張力值為 76.33 ± 0.64 (dyne/cm)當引用修正因子的理論後，所測得的表面張力值和公認值之間的誤差小於 0.3%

在測量桿舉起的過程中，力臂會不斷的改變，經由測量桿角度的測量，在新型的表面張力測定儀中力臂所造成的影響已能晚整的修正。

若使用目前較為精密的光電盤(0.18°)應可以使實驗的結果更為準確。

結論

經由彈性理論所推導出的公式而設計出自動化表面張力測定儀，可以直接測量待測液體的表面張力值，完全擺脫了使用表格的傳統間接測量法。此外相較於目前最常使用的都諾表面張力測定儀，測量環脫離的瞬間可以被準確的測知(其感測方式則是前所為有的新方法)、實驗過程因步進馬達的使用而更為接近準靜狀態且力臂所造成的誤差也完成了修正的工作，可以達到低於 0.3% 的實驗誤差。

由於自動化的完成，整個實驗變得更為省時和有效率，對於實驗的過程和結果分析亦可以做即時的呈現。此外由於實驗儀器和電腦間的連結完全經由電腦的標準連接埠，所以整個表面張力的自動化測量可以擺脫以往介面卡的限制。

參考資料

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A Novel Surface Tension Measuring Method and Apparatus

Introduction

The measurement of surface tension plays an important role in the research of the molecular forces at the interface of liquid and air. Du Noüy's apparatus is the widely used apparatus because of its simplicity and sufficient accuracy.[1] However, through the Du Noüy's apparatus, the surface tension of water (the reference liquid) could not be obtained, the actual moment of the ring detaching from liquid is not detected, the decrease of the moment arm while the lever lifting is not taken into consideration, and the condition of quasi-equilibrium state is difficult to be satisfied by controlling the knob manually. In order to make the measurement of surface tension more accurate and efficient, a PC-based automatic measuring system was designed and set up based on the Du Noüy's apparatus. The drawbacks of the Du Noüy's apparatus described above are removed in the newly created apparatus.

Principle and theory

Principle of Du Noüy surface tensiometer

Photo 1.Setup of Du Noüy surface tensiometer

The basic principle of the Du Noüy's apparatus is based upon the adherence of a ring to the liquid. The torsion of the wire is used to counteract the tension of the liquid film and to break it. If the apparatus has been previously standardized by using water, it gives the surface tension of the liquid by a simple proportion. The equation is as follow：

$$\gamma = \gamma_w \times \frac{\theta}{\theta_w} \qquad (1)$$

where γ (γ_w) is the surface tension of the liquid (water) and θ (θ_w) is the twisted angle of the testing liquid (water), respectively.

Modified surface tensiometer

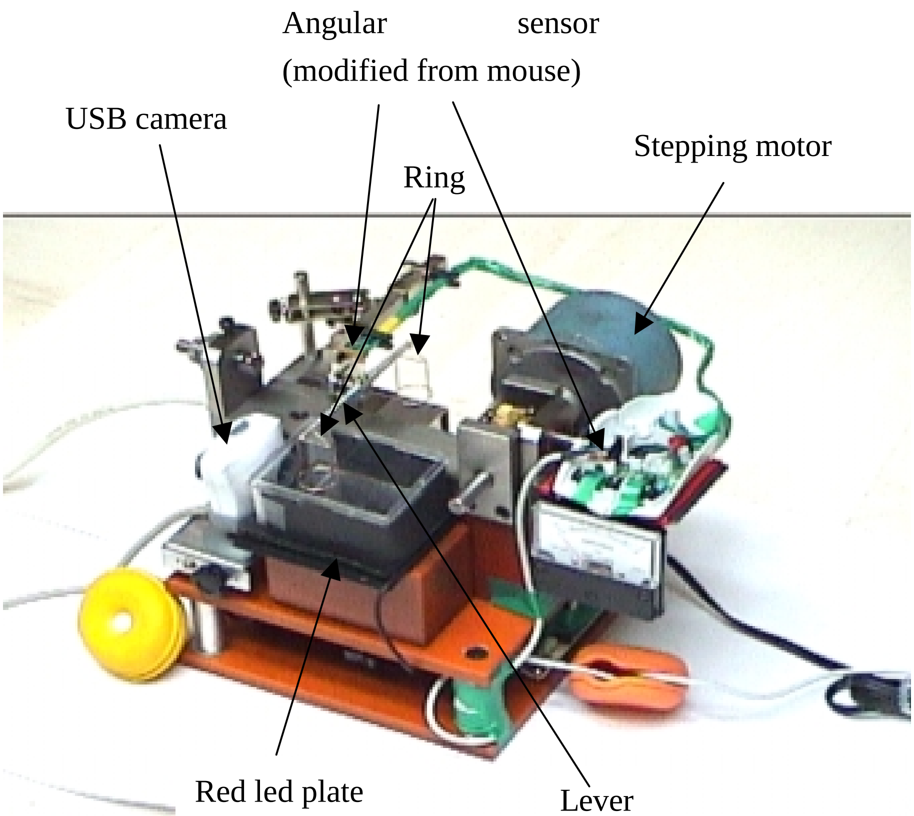

Photo 2.Modified setup of Du Noüy type surface tensiometer

The operational principle of the modified apparatus is based on a formula derived from the theory of elasticity. Consider a circular shaft that is attached to a fixed support at one end as shown in Fig.1 (a). If a torque T is applied to the other end, the shaft will twist, with its free end rotating through an angle ¡í called the twist angle as shown in Fig.1 (b). The specific relation existing between T and ¡í is derived in theory of elasticity as follow :

$$\phi = \frac{T \cdot L}{G \cdot J} \qquad (2)$$

where T is the torque exerted on the shaft, G is the modulus of rigidity or shear modulus of the material, J is the polar moment of inertia of the cross section with respect to its center, and L is the length of the shaft. The relation shows that, the twist angle ¡íis proportional to the torque T applied to the shaft and the torque can be determined by the measurement of the twist angle.

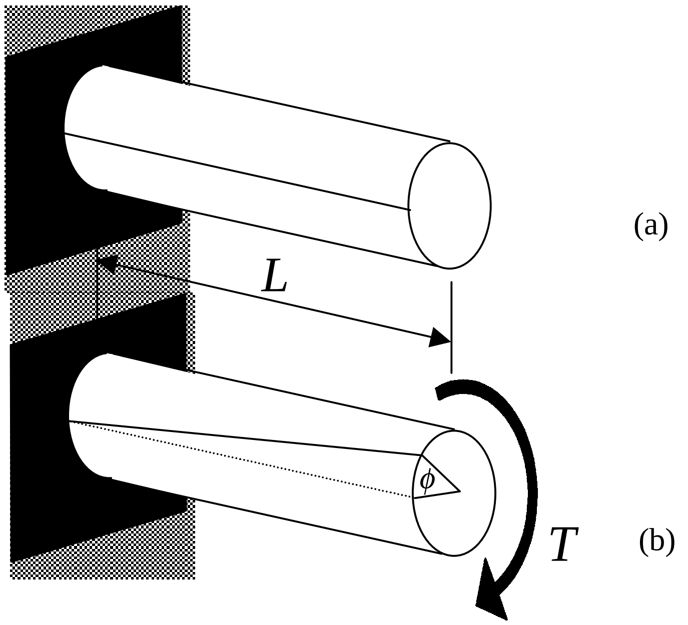

Fig.1 Model of the theory of elasticity

The model of the modified apparatus is demonstrated in Fig.2, where T_1 is the torque from the knob driven by stepping motor, T_2 is the torque induced by the surface tension of the liquid, and ¡í$_B$,¡í$_C$ are the angles of twist relative to the fixed end respectively.

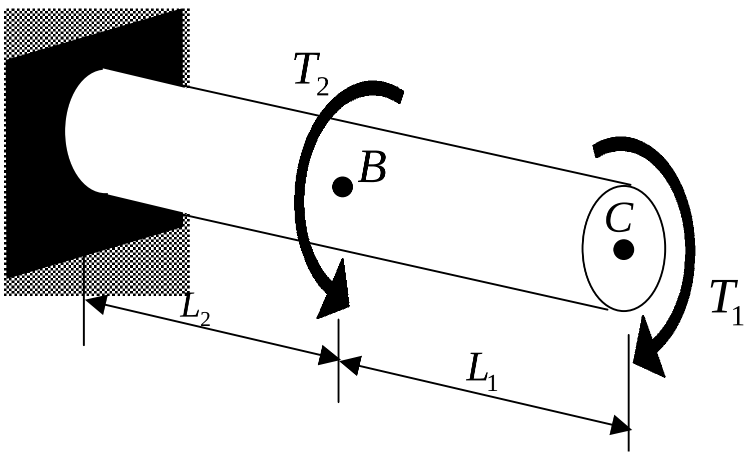

Fig.2 Model of modified apparatus

To show in detail, two free-body diagrams are taken as shown in Fig.3 and Fig.4.

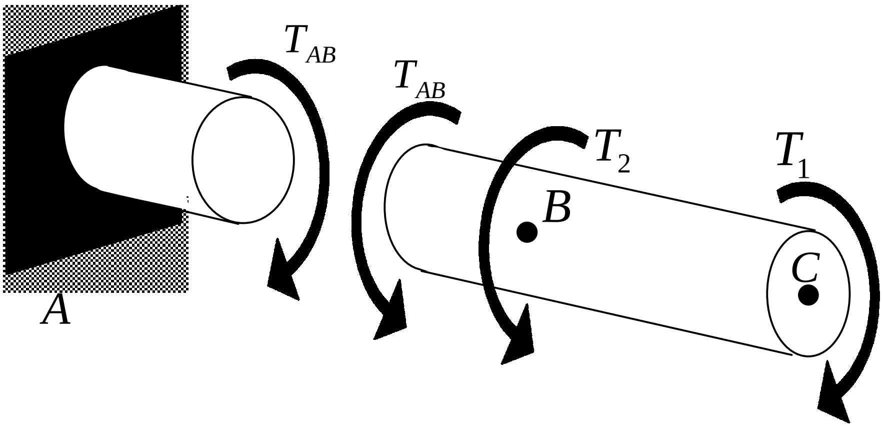

Fig.3 Free-body diagram taken between A and B

In Fig.3, a free-body diagram is taken by passing a section through the wire at any arbitrary location between A and B. T_{AB} represents the internal torque in the section. T_{AB} is obtained by writing that the sum of the torques applied to the right portion is zero.

$$T_{AB} + T_2 - T_1 = 0 \quad (3)$$

According to Eq. (2), the twist angle of the shaft relative to the fixed end may be obtained by writing

$$\phi_B = \frac{T_{AB} L_2}{GJ} = \frac{(T_1 - T_2) L_2}{GJ} \quad (4)$$

In the same way, by passing a section through the wire at any arbitrary location between B and C, the free-body diagram is drawn as shown in Fig.4.

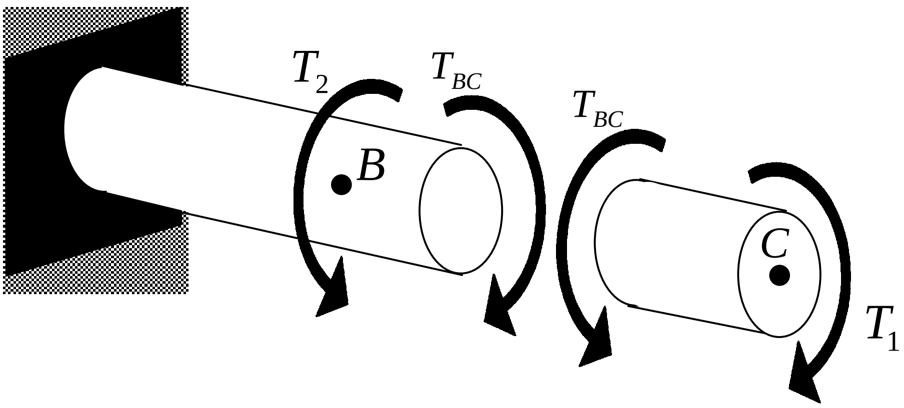

Fig.4 Free-body diagram taken between A and B

Applying the equilibrium equation on the right portion of the wire yields

$$T_{BC} - T_1 = 0 \quad (5)$$

Hence, Eq.(6) is obtained by substituting Eq. (5) into Eq.(2).

$$\phi_{C/B} = \frac{T_{BC} \cdot L_1}{G \cdot J} = \frac{T_1 \cdot L_1}{G \cdot J} \quad (6)$$

Since the twist angle of the wire at C with respect to the fixed end A (ϕ_C) is equal to the sum of the twist angle of B (ϕ_B) and the twist angle of C relative to B ($\phi_{C/B}$), We have

$$\phi_C = \phi_{C/B} + \phi_B \quad (7)$$

Substituting equation (4) and (6) into (7) yields

$$\phi_C = \frac{(T_1 - T_2) \cdot L_2}{G \cdot J} + \frac{T_1 \cdot L_1}{G \cdot J} \quad (8)$$

The two unknowns, T_1 and T_2, can be obtained by solving the simultaneous Eq. (4) and Eq.(8)

$$T_1 = G \cdot J \cdot \frac{\phi_C - \phi_B}{L_1} \quad (9)$$

$$T_2 = G \cdot J \cdot \left(\frac{\phi_C - \phi_B}{L_1} - \frac{\phi_B}{L_2}\right) \quad (10)$$

In Eq.(10), if $L_1 = L_2 = L$ then

$$T_2 = \frac{G \cdot J}{L} \cdot (\phi_c - 2 \cdot \phi_B) = K \cdot (\phi_c - 2 \cdot \phi_B) \quad (11)$$

Once, ϕ_B and ϕ_C is measured, T_2 can be calculated according to Eq. (11).

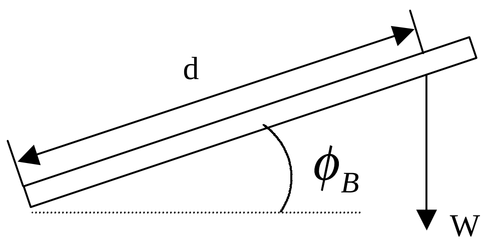

Fig.5 Correction of moment arm

In Fig.5, d represents the length measured from the ring to the center of the torsion wire, W is the force pulling down by the surface tension. Thus,

$$T_2 = W \cdot d \cdot Cos\phi_B \quad (12)$$

$$W = 2\pi \cdot (R_i + R_o) \cdot \gamma \quad (13)$$

using Eqs.(11),(12) and (13) the surface tension γ can be obtained in terms of ϕ_B and ϕ_C and expressed as follow:

$$\gamma = \frac{K \cdot (\phi_C - 2 \cdot \phi_B)}{2\pi \cdot (R_i + R_o) \cdot d \cdot Cos\phi_B} = K_1 \cdot \frac{\phi_C - 2 \cdot \phi_B}{Cos\phi_B} \quad (14)$$

where γ is the surface tension of the liquid, R_i,R_o represent the inner and outer radii of the ring and K,K_1 are constants.

Equipment

6.Detaching ring and liquid holder
7.Stepping motor and power supply
8.Steel wire
9.Notebook PC (with LPT cable)
10.I-com (USB Camera)

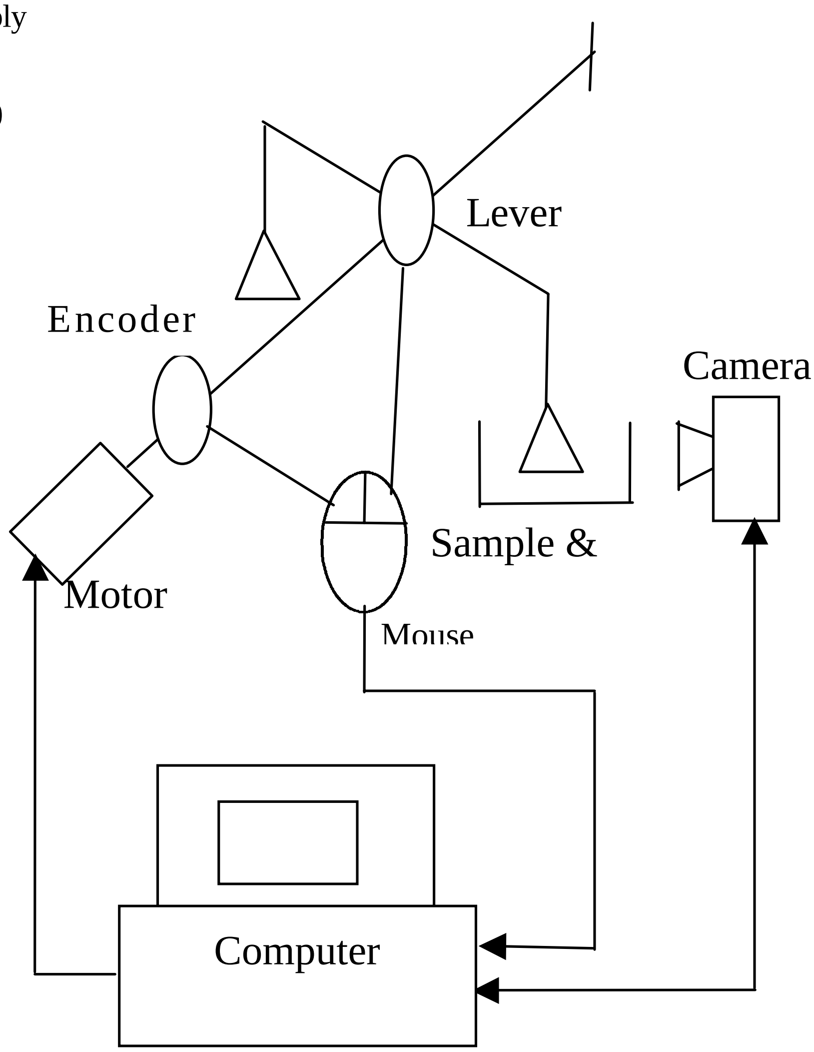

Fig.6 Construction of modified apparatus

Fig.6 shows the construction of the modified apparatus. The additional key components implemented upon the base structure of the Du Noüy's apparatus are demonstrated as follows:

1.A stepping motor is mounted on the thumb knob of the Du Noüy's apparatus. The stepping motor is driven via the parallel port of the PC. The main function of the stepping motor is to supply a steady and constant torque to turn the knob. In this way, not only the quasi-equilibrium state could be achieved but also the aim of automation of measurement was fulfilled.

2.A mouse of PC was modified to serve as two encoders. One set of the position sensor of the mouse was removed from the pc board and remounted at the middle of the lifting lever, which supported the testing ring.

3.A PC camera was mounted by the side of the ring to monitor the behavior of the liquid film in the process of experiment.

Materials and Methods

A.Samples

1.Water

2.Alcohol

B.Measuring method

The schematic functional block diagram of the automatic surface tension measurement system is shown as Fig.7

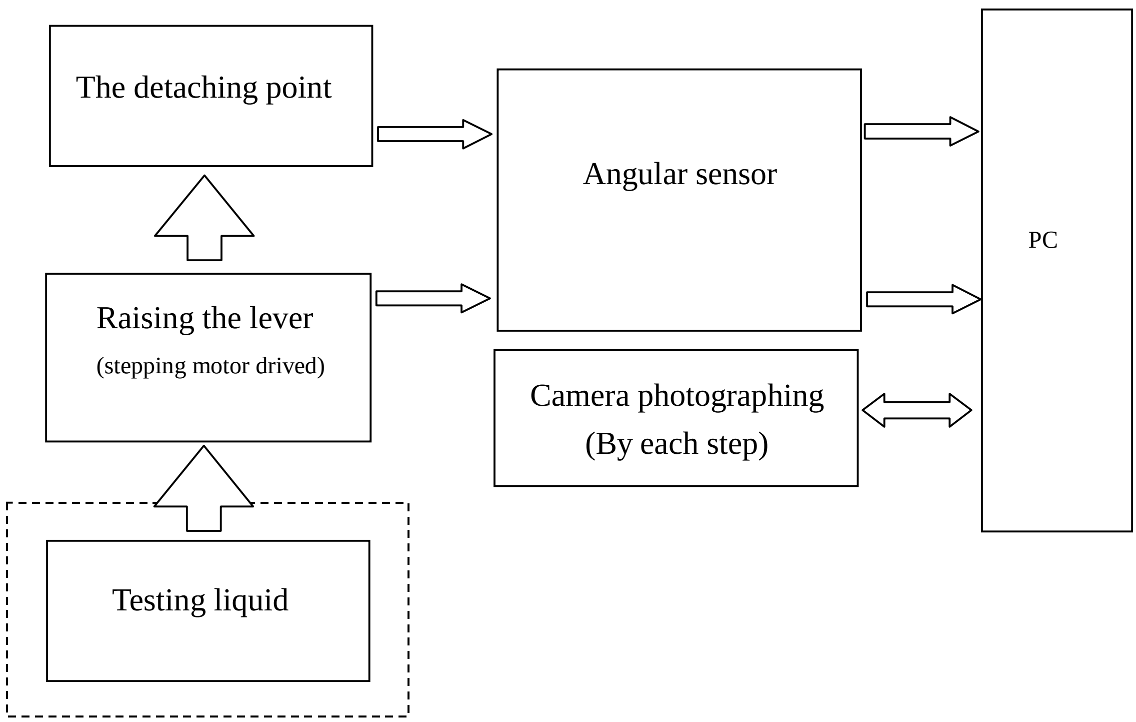

Fig. 7 Schematic functional block diagram of the automatic surface tension measurement system

As the experiment began, the lever will be lifted by the stepping motor until the ring detaching from the liquid. During the process, pictures were taken at each pulling step synchronized with the stepping motor. A program written in Visual Basic based on the image-processing algorithm analyzed the pictures in the real time. In the program, the red layer image was extracted from the color image captured by the USB camera, and then transformed into binary image with a suitable threshold in each pulling step. The four original images, red layer images and processed binary images during the experiment in sequence are shown in Fig.8. Let D be the total number of pixels with different binary value between two successive binary images. It is obvious that D_3 increases abruptly as the film breaks.

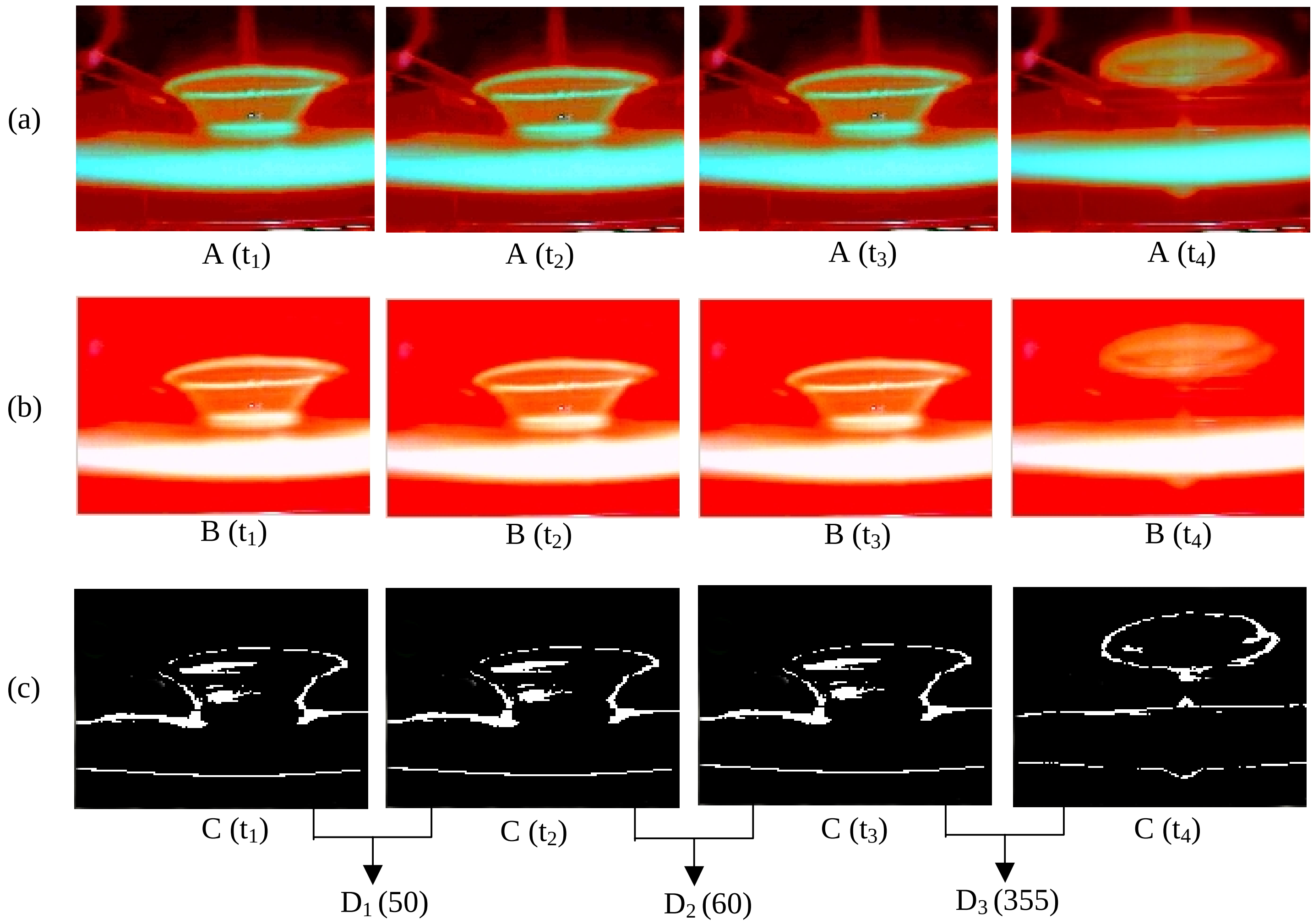

Fig.8 (a)Original images shot by the USB camera, (b)Red layer images (c)Quantized binary images

In this way, the detaching point of the ring and testing liquid could be detected precisely. At the moment of detaching, the twist angles of ϕ_B and ϕ_C were obtained via the modified encoder. With the twist angles ϕ_B and ϕ_C, the surface tension of the liquid is calculated based on Eq.(14).

Further more, the experiment can be performed several times automatically by simply pressing a button of the mouse. The average value and standard deviation were calculated as soon as the experiment finished. Thus, the automation of measurement is fulfilled.

Results and discussion

A. Comparison between modified apparatus and Du Noüy's apparatus

The relative measuring method (the same as the method of Du Noüy surface tensiometer) can be also performed by the modified apparatus.

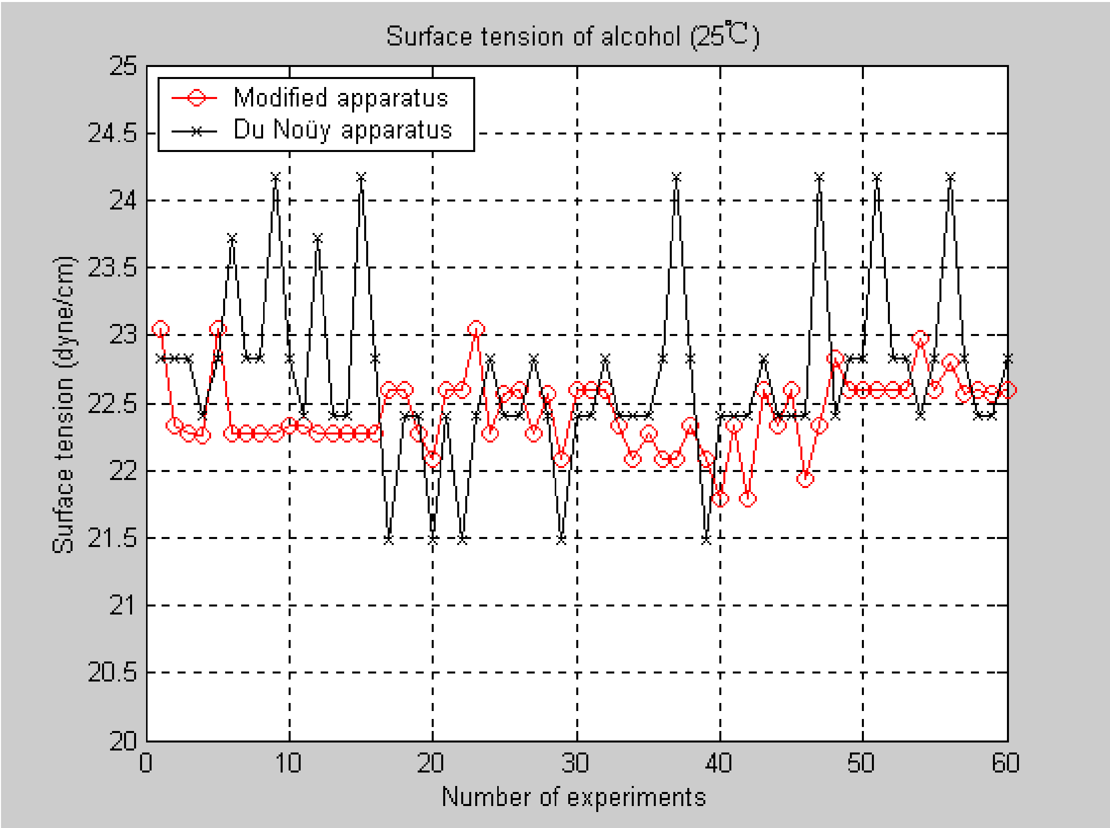

Fig.9 Comparison between Du Noüy apparatus and Modified apparatus (in relative way)

4.Fig.9 shows excellent results of the 60 experimental data point and clearly outperforms the empirical work of the Du Noüy's apparatus. The lower standard deviation is reached by the more precision measurement of the angle of the unfixed side.

5.The experimental data of the modified apparatus lies between 23.0 and 21.8, which are included in the range of Du Noüy's (between 24.2 and 21.5). The reasons for this better result are that the film breaking point can be accurately captured and the angle can be measured more precisely by the higher resolution angular sensor.

6.The surface tension of alcohol measured by the modified apparatus is 22.42 ± 0.28 (dyne/cm) and that of measured by the Du Noüy's apparatus is 22.7 ± 0.7 (dyne/cm). The mean value is closer to the standard value of alcohol (22.39 dyne/cm).

B. Results of the modified apparatus

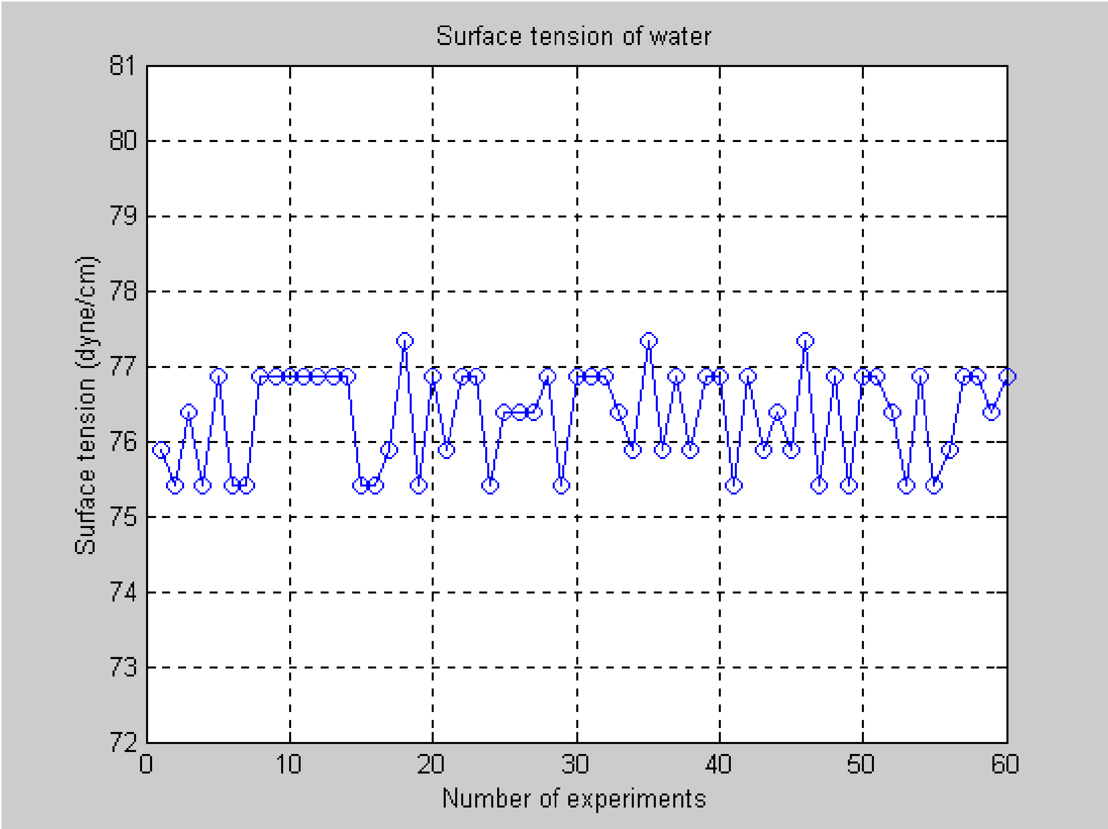

Fig.10 Surface tension measured by the modified apparatus

4.The surface tension measured by the modified apparatus is 76.33 ± 0.64 (dyne/cm) as shown in Fig.10 Incorporating the correcting factor[2,3] into the results, an error between the measured value and the accepted value is within 0.3¢H.

5.The moment arm of the pulling force is changing continuously as the lever gradually rotates about the torsion wire. By measuring the angle of the lever, the decrease of the moment arm of force is mathematically corrected in the modified apparatus.

6.The film breaking point is grabbed accurately through recorded images by USB camera and this is another key factor to improve the accuracy.

While performing the experiment a phenomenon is observed. The lifting ring was accelerated before the film broke and this caused unstable factor that result in errors. The resolution of the angular sensor (modified by the mouse) implemented in the modified apparatus is 0.9°. However, the accuracy of the surface tension can be further improved by using a commercial available encoder with resolution of 0.18°.

Conclusions

A novel surface tensiometer was invented in this project. By introducing the theory of elasticity, the surface tension of the unknown liquid can be measured without comparing with the standard liquid. In contrast to the existing method, the film breaking point can be accurately grabbed by using a USB camera; this has never been done by any former researchers. Furthermore, the condition of quasi-equilibrium state was reached through the application of the stepping motor and the variation of the moment arm owing to the rotation of the lifting lever was corrected. With the proposed apparatus, the accuracy within 0.3% can be obtained.

The automation of measurement is fulfilled via the application of computer. Repeated experiments may be conducted automatically and the mean and standard deviation of the measurements may be calculated. Rather than focusing on tedious methods of gathering data, researchers can focus on results and concepts. Another remarkable feature of this project is that the communication ports between the instrument and computer are all the standard devices of PC. Since no special interface card is needed, the innovated apparatus works perfectly even on a notebook PC.

References

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