The Discovery and Ap pli ca tion of C-Trans for ma tion

Abstract In this study, a new geo met ric trans for ma tion based on a tri an gle and a fixed point is Introduced. We call this the C-transformation. It is non-isometric and non-isogonal.Many curves can be Re-in ves ti gat ed and many extensions can be proved under C-transformation.Here is the main result; (1)The Euler line is determined by the C-transformation. (2)Conics aregenerated from lines under the C-transformation.The relative po si tion of the straight line and the circumscribed ellipse de ter mines whether the conics are circles or ellipses or Parabolas or hyperbolas. A new synthetic construction with the rulers and compasses of conics pass ing through five points is discovered.(3) The family of deltoid, including hypotrochoid and the ex ten sion of hypotrochoid with same nodes,are gen er at ed from circles with the centroid of the triangle as center under the C-transformation. This leads to a new construction of the generalized deltoid passing through five points.(4) The family of strophoid and the family of folium of Descartes are generated from circles with the midpoint of the base of the triangle as center under the C-transformation.In addition, these two families coincide under C-transformation. (5) The family of cissoid are generated from a tri an gle moving along a line under the C-transformation. 1.Motivation

Experiments carried out with the computer indicate that there is always a new geometric transformation and its duality based on a triangle and a fixed point. It is non-isometric, non-isogonal and more complicated than many other well-known transformations. Many curves can be re-investigated and many extensions can be obtained under this transformation.

2.Purpose 3.Procedures

PURPOSE

1. By means of GSP and MAPLE V, we will investigate properties of the new transformation and its duality.
2. Re-investigate Euler line, conjugate Euler line, conics, deltoid, strophoid, folium of Descartes and cissoid under this transformation and attempt to obtain some extensions.
3. Find the parametric equations for each family of many curves. This new equation will show how the family changes continuously.

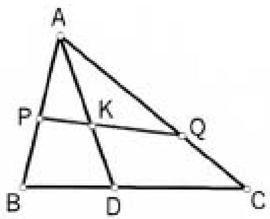
PROCEDURES

1. Fundamental theorems

In this section, the existence theorem of a new geometric transformation based on a triangle and a fixed point will be proved. From now on, $\overrightarrow{AB}$ means the directed line segment. In $\triangle ABC$, K is a fixed point. If $\overleftrightarrow{AK}, \overleftrightarrow{BK}$ and $\overleftrightarrow{CK}$ intersect $\overleftrightarrow{BC}, \overleftrightarrow{AC}$ and $\overleftrightarrow{AB}$ at D, E and F respectively with $\alpha = \frac{\overrightarrow{AF}}{\overrightarrow{FB}}$ and $\beta = \frac{\overrightarrow{BD}}{\overrightarrow{DC}}$, then K is uniquly determined by α, β and denoted by $K\left[\alpha, \beta, \frac{1}{\alpha\beta}\right]$. The following theorem

Theorem of collinearity In $\triangle ABC$, let $K[\alpha, \beta, \gamma]$ be a fixed point. Assume that P and Q lie on $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$ respectively with $x = \frac{\overrightarrow{BP}}{\overrightarrow{PA}}$ and $y = \frac{\overrightarrow{CQ}}{\overrightarrow{QA}}$. Then P, K and Q are collinear if and only if

$$x + \beta y = \frac{1}{\alpha}$$

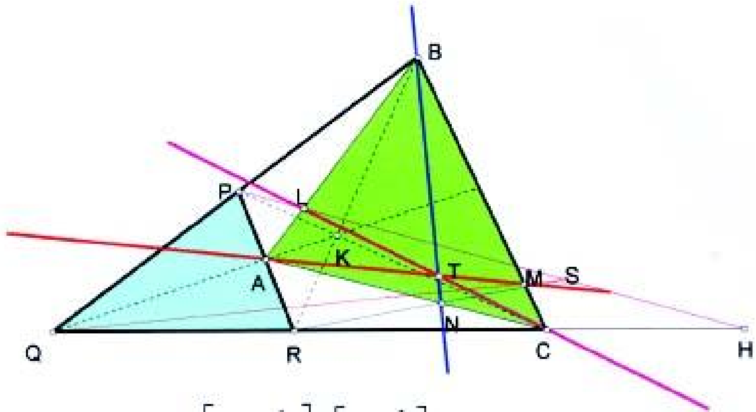


Corollary In $\triangle ABC$, let R lie on $\overleftrightarrow{BC}$ with $y = \frac{\overrightarrow{BR}}{\overrightarrow{RC}}$. Assume that $P\left[a, b, \frac{1}{ab}\right]$ and $Q\left[e, f, \frac{1}{ef}\right]$. Then P, Q and R are collinear if and only if

$$a(b - y) = e(f - y)$$

Next is the fundamental theorem of the new transformation.

Theorem 1 In $\triangle PQR$, let K be a fixed point. Let A, B and C lie on $\overleftrightarrow{PR}$, $\overleftrightarrow{PQ}$ and $\overleftrightarrow{QR}$ respectively and $\overleftrightarrow{PC}$, $\overleftrightarrow{QA}$ and $\overleftrightarrow{RB}$ be concurrent at K. For any point S, if $\overleftrightarrow{SP}$, $\overleftrightarrow{SQ}$ and $\overleftrightarrow{SR}$ intersect $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{AC}$ at L, M and N respectively, then $\overleftrightarrow{AM}$, $\overleftrightarrow{BN}$ and $\overleftrightarrow{CL}$ are concurrent.



Proof Let $K\left[\alpha,\beta,\frac{1}{\alpha\beta}\right], S\left[t,u,\frac{1}{tu}\right]$

In $\triangle PQR$, since B, L and A are collinear,

$\therefore \frac{1}{\alpha}+\frac{u}{\alpha\beta}=\frac{u+1}{h}$, where $h=\frac{\overrightarrow{PL}}{\overrightarrow{LH}}$

$\therefore h=\frac{\alpha\beta(u+1)}{\beta+u}, L\left[\frac{\alpha\beta}{\beta+u},u,\frac{\beta+u}{\alpha\beta u}\right]$

Then $T\left[x,y,\frac{1}{xy}\right]$ lies on $\overleftrightarrow{CL}$ if and only if

$$\frac{\alpha\beta}{\beta+u}(u-\beta)=x(y-\beta) \quad \ldots\ldots(1)$$

Similarly, T lies on $\overleftrightarrow{AM}$ if and only if

$$\frac{\beta tu}{tu+\alpha\beta}\left(\frac{1}{tu}-\frac{1}{\alpha\beta}\right)=y\left(\frac{1}{xy}-\frac{1}{\alpha\beta}\right) \quad \ldots\ldots(2)$$

T lies on $\overleftrightarrow{BN}$ if and only if

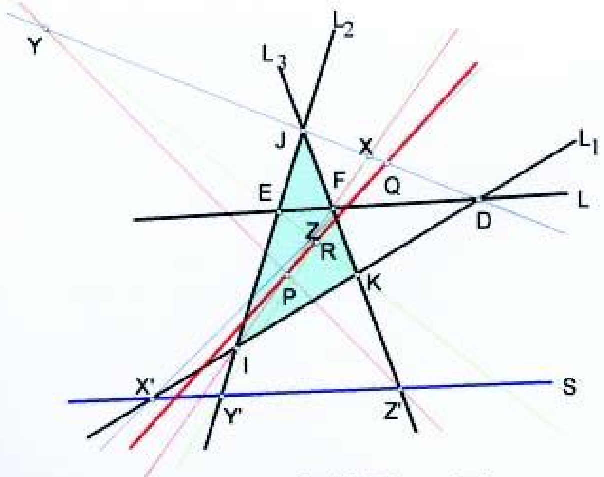
$$\frac{t-\alpha}{\beta(\alpha+t)}=\frac{x-\alpha}{xy} \quad \ldots\ldots(3)$$

Since (1), (2), (3) have same solution $x = \frac{\alpha\beta + tu}{\beta + u}, y = \frac{\beta u(t + \alpha)}{\alpha\beta + tu}$

$\therefore \overleftrightarrow{AM}, \overleftrightarrow{BN}$ and $\overleftrightarrow{CL}$ are concurrent.

Next theorem is the dual theorem of theorem 1.

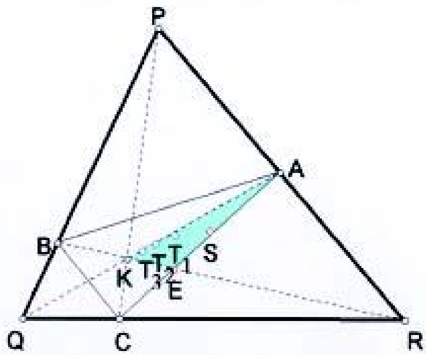
Dual theorem Assume that L_1, L_2 and L_3 are lines and L_1, L_2, L_3 meets L_2, L_3, L_1 at I, J, K respectively. Let L be a line and L meet L_1, L_2, L_3 at D, E, F respectively. Let $\overleftrightarrow{IF}$, $\overleftrightarrow{JD}$ and $\overleftrightarrow{KE}$ meet $\overleftrightarrow{JD}$, $\overleftrightarrow{KE}$ and $\overleftrightarrow{IF}$ at X, Y, and Z respectively. For any line S, if S meets L_1, L_2 and L_3 at X', Y' and Z' respectively, then the intersection points of $\overleftrightarrow{IF}$ and $\overleftrightarrow{YZ'}$, $\overleftrightarrow{JD}$ and $\overleftrightarrow{ZX'}$, $\overleftrightarrow{KE}$ and $\overleftrightarrow{XY'}$ are collinear.



Definition 1 In theorem 1, let $\overleftrightarrow{AM}, \overleftrightarrow{BN}$ and $\overleftrightarrow{CL}$ be concurrent at T. Then T is the C-transformation of S under K in $\triangle PQR$. Denote T as T=S(K).

Corollary 1 S(K)=K(S)

Corollary 2 For any point S, let T_1 =S(K), $T_2 = T_1$ (K), $T_{n+1}=T_n$ (K), $n \ge 2$. Then the sequence $\{T_n\}$ approaches K.



Proof Let S be an interior point of $\triangle$AKE. Then $S\left[P\,l, \frac{m}{pq}, \frac{q}{lm}\right]$,

where $l=\frac{\alpha+1}{\alpha\beta+1}$, $m=\frac{\alpha(\beta+1)}{\alpha+1}$, 0<p<1, q>1, pq<1

Then S(K) is

$$T_1\left[\frac{(pq+1)(\alpha+1)}{(q+1)(\alpha\beta+1)}, \frac{\alpha(q+1)(\beta+1)}{q(p+1)(\alpha+1)}, \frac{q(p+1)(\alpha\beta+1)}{\alpha(pq+1)(\beta+1)}\right]$$

$$\therefore T_1\left[\frac{(pq+1)l}{q+1}, \frac{(q+1)m}{q(p+1)}, \frac{q(p+1)}{pq+1}\cdot\frac{1}{lm}\right]$$

where $0<\frac{pq+1}{q+1}<1$, $\frac{q(p+1)}{pq+1}>1$, $0<\frac{q(p+1)}{q+1}<1$

Similarly, we have

$$T_1\left[p_1 l, \frac{m}{p_1 q_1}, \frac{q_1}{lm}\right], \quad p_1=\frac{pq+1}{q+1}, q_1=\frac{q(p+1)}{pq+1}$$

$$T_2\left[p_2 l, \frac{m}{p_2 q_2}, \frac{q_2}{lm}\right],\ p_2 = \frac{p_1 q_1 + 1}{q_1 + 1}, q_2 = \frac{q_1(p_1 + 1)}{p_1 q_1 + 1}$$

$$T_n\left[p_n l, \frac{m}{p_n q_n}, \frac{q_n}{lm}\right],\ p_n = \frac{p_{n-1} q_{n-1} + 1}{q_{n-1} + 1}, q_n = \frac{q_{n-1}(p_{n-1} + 1)}{p_{n-1} q_{n-1} + 1}$$

where $0 < p_n < 1$, $q_n > 1$. Consider $\{p_n\}, \{q_n\}$

$$\because p_n - p_{n-1} = \frac{p_{n-1} q_{n-1} + 1}{q_{n-1} + 1} - p_{n-1} = \frac{1 - p_{n-1}}{q_{n-1} + 1} > 0$$

$\therefore \{p_n\}$ is increasing.

$$\because q_n - q_{n-1} = \frac{q_{n-1}(p_{n-1} + 1)}{p_{n-1} q_{n-1} + 1} - q_{n-1} = \frac{p_{n-1} q_{n-1}(1 - q_{n-1})}{p_{n-1} q_{n-1} + 1} < 0$$

$\therefore \{q_n\}$ is decreasing.

Since $p_n < 1, q_n > 1$, so $\{p_n\}$ and $\{q_n\}$ have limits.

Let $\lim_{n\to\infty} p_n = I$, $\lim_{n\to\infty} q_n = J$

$$\therefore I = \lim_{n\to\infty} p_n = \lim_{n\to\infty} \frac{p_{n-1} q_{n-1} + 1}{q_{n-1} + 1} = \frac{IJ + 1}{J + 1}$$

$\therefore I = 1$, $J = 1$

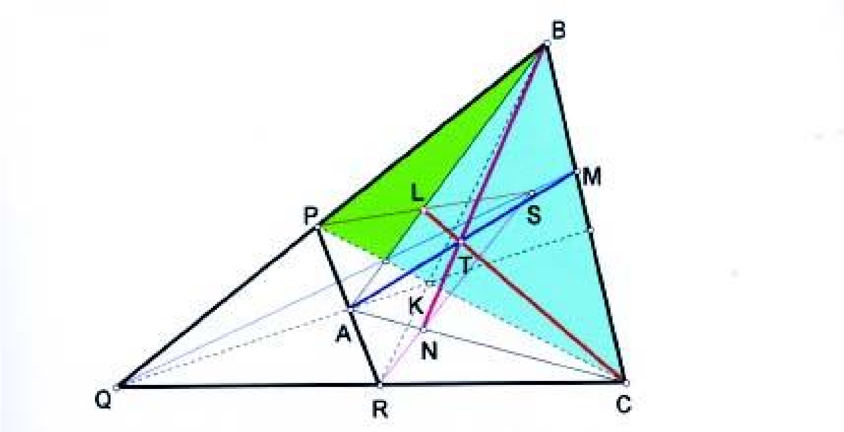
$\therefore T_n\left[p_n l, \frac{m}{p_n q_n}, \frac{q_n}{lm}\right]$ approaches $K\left[1, m, \frac{1}{lm}\right]$

The C-transformation is characterized in the next corollary.

Corollary 3 Assume that K is an exterior point of $\triangle PQR$. Let $\overleftrightarrow{PC}, \overleftrightarrow{QA}$ and $\overleftrightarrow{RB}$ be concurrent at K. Let S be an interior point of quadrilateral BPRC. Let L lie on $\overline{AB}$ with

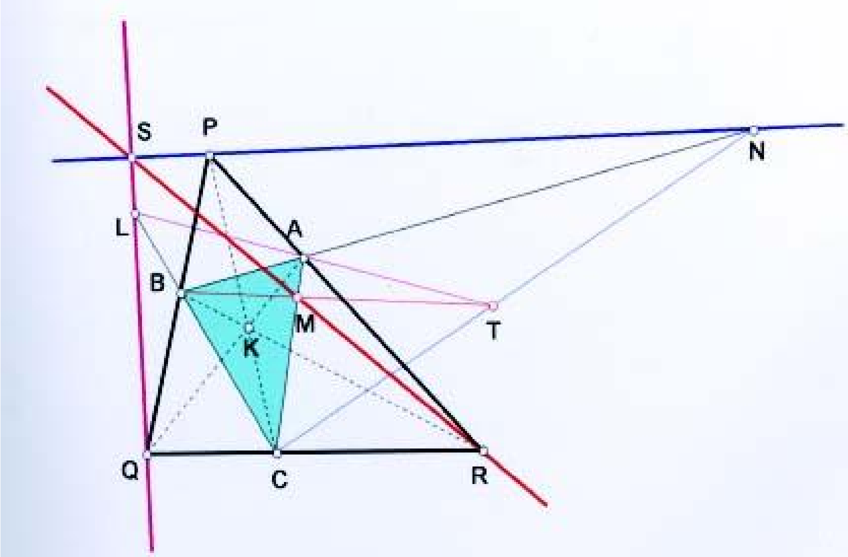
$$\sin\angle BCP \cdot (\cot\angle BCL \cdot \sin\angle ABC + \cos\angle ABC) = \sin\angle BPC \cdot (\cot\angle BPS \cdot \sin\angle ABP + \cos\angle ABP)$$

Then $\overleftrightarrow{CL}, \overleftrightarrow{AM}$ and $\overleftrightarrow{BN}$, having similar properties, are concurrent at S(K).



Next theorem is the converse of theorem 1.

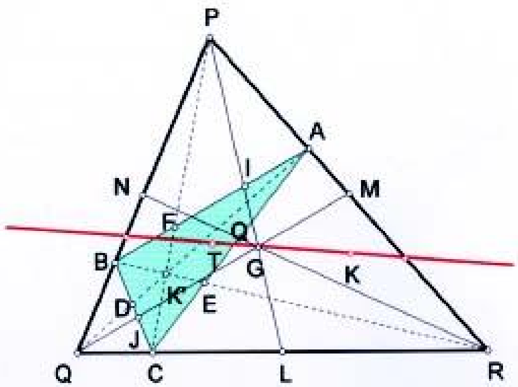
Theorem 2 In $\triangle PQR$, let K be a fixed point. Let $\overleftrightarrow{PC}$, $\overleftrightarrow{QA}$ and $\overleftrightarrow{RB}$ be concurrent at K. For any point T, if $\overleftrightarrow{TA}$, $\overleftrightarrow{TB}$ and $\overleftrightarrow{TC}$ intersect $\overleftrightarrow{BC}$, $\overleftrightarrow{AC}$ and $\overleftrightarrow{AB}$ at L, M and N respectively, then $\overleftrightarrow{PN}$, $\overleftrightarrow{QL}$ and $\overleftrightarrow{RM}$ are concurrent at S and T=S(K).



2. Euler line and conjugate Euler line

In this section, for $K[a,b,c]$, let $K'\left[\frac{1}{a},\frac{1}{b},\frac{1}{c}\right]$ be the reciprocal point of K.

Theorem 3 In $\triangle PQR$, let G be the centroid. Assume that K is an arbitrary point and $T=K'(G)$. Then K, G and T are collinear and $\overrightarrow{KG}=2\overrightarrow{GT}$



Proof In $\triangle PQR$, let $K\left[\frac{1}{\alpha},\frac{1}{\beta},\alpha\beta\right], \alpha>1, \beta<1, \alpha\beta<1$

By theorem of collinearity

$$\frac{\overline{BI}}{\overline{IA}}=\frac{\frac{1}{\alpha\beta}+1}{\frac{1}{\alpha}+1}=\frac{\alpha\beta+1}{\beta(\alpha+1)} \quad , \quad \frac{\overline{CJ}}{\overline{JB}}=\frac{\alpha+1}{\frac{1}{\beta}+1}=\frac{\beta(\alpha+1)}{\beta+1}$$

$$\frac{\overline{AQ}}{\overline{QC}}=\frac{\beta+1}{\alpha\beta+1}$$

$$\therefore \text{In } \triangle ABC, \quad T\left[\frac{\beta(\alpha+1)}{\alpha\beta+1},\frac{\beta+1}{\beta(\alpha+1)},\frac{\alpha\beta+1}{\beta+1}\right]$$

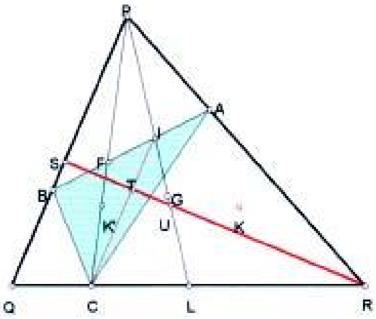


Figure 1

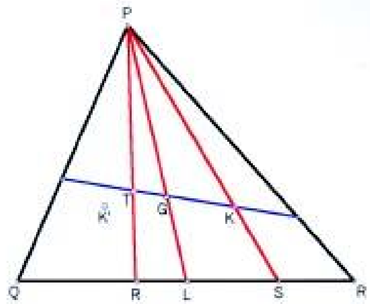


Figure 2

In figure 2, $G[1,1,1]$, $K\left[\frac{1}{\alpha},\frac{1}{\beta},\alpha\beta\right]$

$$\because \quad \begin{cases} x+y=1 \\ x+\frac{1}{\beta}y=\alpha \\ x+\frac{\beta(\alpha+1)}{\alpha\beta+1}y=\frac{\beta+1}{\alpha\beta+1} \end{cases}$$

has solution $x=\frac{\alpha\beta-1}{\beta-1}>0,\quad y=\frac{\beta(1-\alpha)}{\beta-1}>0,$

$$\therefore \quad \frac{\overline{TG}}{\overline{GK}}=\frac{\frac{\overline{RL}}{\overline{LS}}\left(\frac{\overline{SK}}{\overline{KP}}+1\right)}{\frac{\overline{RT}}{\overline{TP}}+1}=\frac{1}{2} \qquad \therefore \overline{KG}=2\overline{GT}$$

Corollary 4 Assume that H, G and O are the orthocenter, centroid and circumcenter of a triangle respectively and W is the center of the nine points circle. Then $O=H'(G)$ and $W=O'(G)$.

Corollary 5 Assume that I and G are the incenter and centroid of a triangle. Then the conjugate Euler line is determined by I, G and $I'(G)$.

3. Conics

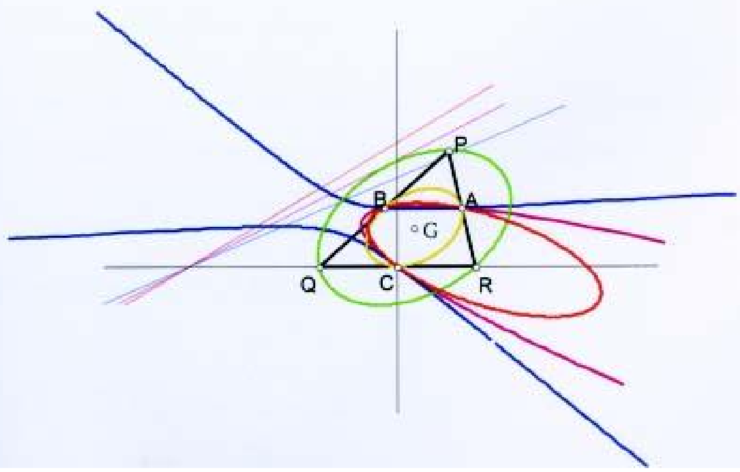
In this section, conics will be generated from straight lines under C-transformation. In this way, a new construction with rulers of conics passing through five distinct points will be obtained. From now on, the ellipse passing through P, Q and R and having the centroid of △PQR as symmetric center will be named the circumscribed ellipse of △PQR. The circumscribed ellipse will completely determine whether the conics are circles or ellipses or parabolas or hyperbolas.

Theorem 4 In △PQR, let A, B and C be the midpoints of sides and G be the centroid. Assume that L is an arbitrary line and L does not pass through P, Q and R. Let V be the curve of C-transformation of points in L under G. Then V is:

(1) a circle or an ellipse if L and the circumscribed ellipse of △PQR are disjoint;

(2) a parabola if L is tangent to the circumscribed ellipse of △PQR;

(3) a hyperbola if L is secant to the circumscribed ellipse of △PQR.



Proof Let P(2b, 2c), Q(-2a, 0), R(2a, 0), A(a+b, c), B(b-a, c), C(0, 0)

Let L be a line passing through (f,0) and having slope m with $m \neq 0$,

$$m \neq \frac{2c}{2b-f}$$

For any $S(t, m(t-f))$ in L , let T(x,y)=S(G). Then

$$(-c^2fm + 2bc^2m - 2c^3)x^2 + (8a^2cm + 2bcfm - 4b^2cm + 4bc^2)xy$$
$$+ (-b^2fm - 3a^2fm + 2b^3m - 10a^2bm + 2a^2c - 2b^2c)y^2 - 8a^2c^2mx$$
$$+ (4a^2cfm + 8a^2bcm)y = 0$$

It is a conic section passing through A, B and C.

For any S(t,u), let T(x,y)=S(G). Then

$$x = \frac{-4a^2cu(2c-u)}{3a^2u^2 + b^2u^2 - 4a^2cu + c^2t^2 - 4a^2c^2 - 2bctu}$$
$$y = \frac{-4a^2u(2bc + ct - 2bu)}{3a^2u^2 + b^2u^2 - 4a^2cu + c^2t^2 - 4a^2c^2 - 2bctu}$$

Then the circumscribed ellipse of $\triangle$PQR is

$$c^2x^2 - 2bcxy + (3a^2 + b^2)y^2 - 4a^2cy - 4a^2c^2 = 0$$

The solution of L and the circumscribed ellipse completely determines whether conics are circles or ellipses or parabolas or hyperbolas.

Corollary 6 The C-transformation of infinite point of L lies on the circumscribed ellipse of $\triangle$ABC.

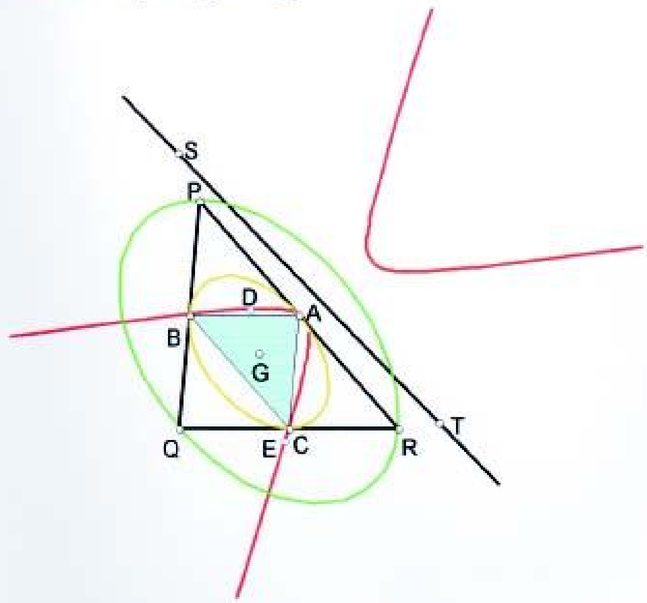
Proof Let t approach infinity, then T(x,y) is

$$x = \frac{-4a^2m(c-2bm)}{3a^2m^2+b^2m^2+c^2-2bcm}$$

$$y = \frac{4a^2cm^2}{3a^2m^2+b^2m^2+c^2-2bcm}$$

Then $c^2x^2-2bcxy+(3a^2+b^2)y^2-4a^2cy=0$ is the circumscribed ellipse of $\triangle ABC$

Corollary 7 Given five distinct points, with no three collinear, there is always a conic section passing through them.



Proof Given A, B, C, D and E. Let $\triangle PQR$ be a triangle with A, B and C the midpoints of sides and G the centroid. Let D=S(G), E=T(G). Then the curve of C-transformation of $\overleftrightarrow{ST}$ is a conic section passing through A, B, C, D and E.

4. Family of deltoid

In this section, the family of deltoid, including hypotrochoid and extensions of hypotrochoid with same nodes, will be generated from circles. In this way, a construction of generalized deltoid passing through five distinct points will be proved.

Theorem 5 Assume that $\triangle PQR$ is equilateral. Let G be the centroid. Let H be a circle with G as center. Then the curves of C-transformation of points in H under G form a family of deltoid, including hypotrochoid and extensions of hypotrochoid with same nodes.

Proof Let $P(r,\sqrt{3}r), Q(-2r,0), R(r,-\sqrt{3}r), S(2l\cos\theta, 2l\sin\theta)$

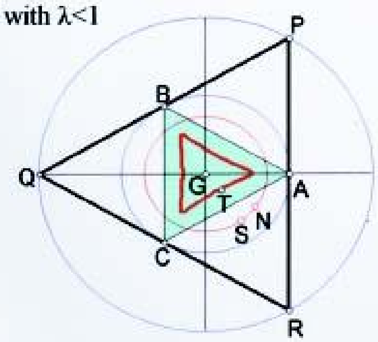
Then S(G)=T(x,y) with

$$\begin{cases} x = \dfrac{-rl(l\cos 2\theta + r\cos\theta)}{l^2 - r^2} \\ y = \dfrac{rl(l\sin 2\theta - r\sin\theta)}{l^2 - r^2} \end{cases}$$

Let $\dfrac{r}{l} = m$, then

$$\begin{cases} x = \dfrac{r}{m^2 - 1}(m\cos\theta + \cos 2\theta) \\ y = \dfrac{r}{m^2 - 1}(m\sin\theta - \sin 2\theta) \end{cases}$$

(1) m>2 hypotrochoid with $\lambda<1$



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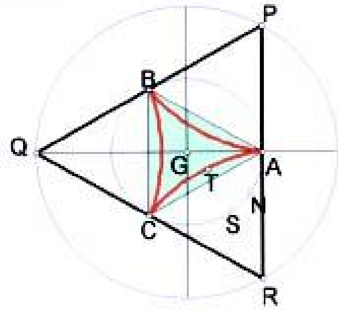
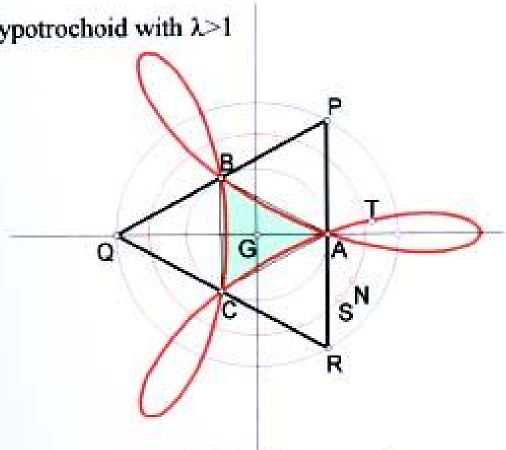
(2) m = 2 deltoid of the inscribed circle of $\triangle PQR$

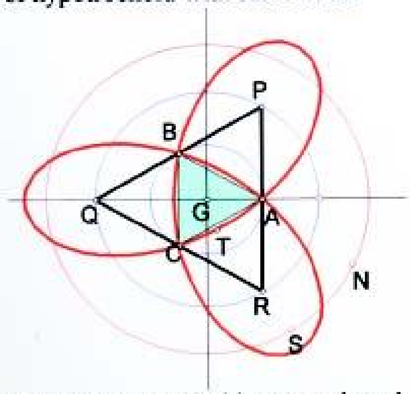
$$\begin{cases} x = \frac{r}{3}(2\cos\theta + \cos 2\theta) \\ y = \frac{r}{3}(2\sin\theta - \sin 2\theta) \end{cases}$$



(3) $1<m<2$ hypotrochoid with $\lambda>1$



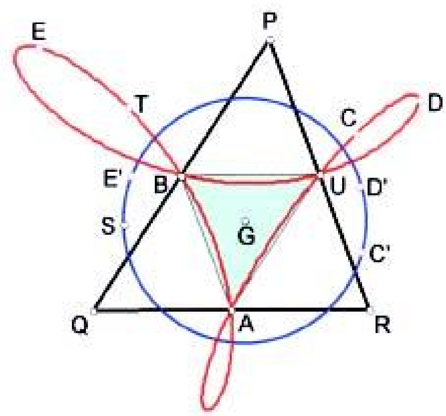
(4) $m<1$ extension of hypotrochoid with same nodes



(5)$m \to 0$ the extension of hypotrochoid approaches the inscribed circle of $\triangle PQR$

Definition 2 In $\triangle PQR$, let G be the centroid. Let H be a circle with G as center. Then the curve of C-transformation of points in H under G is named a generalized deltoid.

Corollary 8 Given five distinct points, with no three collinear, there is always a generalized deltoid passing through them.



5. Some famous curves

In this section, families of strophoid, folium of Descartes and cissoid are shown as the results of the C-transformation. In addition, the strophoid and the folium of Descartes are two special curves of a family under C-transformation.

Theorem 6 Assume that $\triangle PQR$ is isosceles with base $\overline{PQ}$. Let G be the centroid. Let H be a circle with the midpoint of base as center and passing through R. Then the curves of C-transformation of points in H under G form a family of strophoid and folium of Descartes.

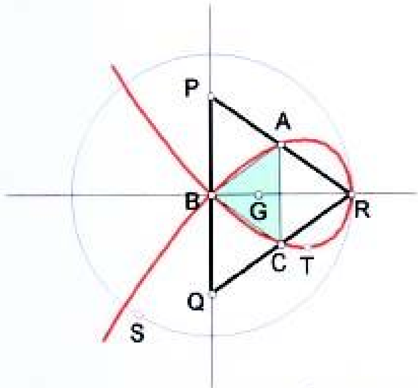
Proof Let P(0, 2b), Q(0, -2b), R(2a, 0), S(2acos θ, 2asin θ).

Then S(G)=T(x, y) with

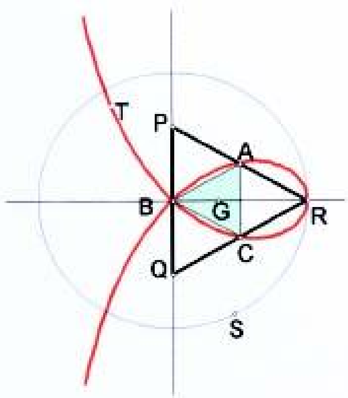
$$\begin{cases} x = \dfrac{-4ab^2\cos\theta}{a^2\cos\theta - 3b^2\cos\theta + a^2 - b^2} \\ y = \dfrac{4ab^2\sin\theta\cos\theta}{-a^2 + a^2\cos^2\theta + 2b^2\cos\theta - 3b^2\cos^2\theta + b^2} \end{cases}$$

$$\therefore\quad y^2 = \frac{x^2(2ab^2 - b^2x)}{2ab^2 + (a^2 - 2b^2)x} = \frac{x^2(2a - x)}{2a + (t^2 - 2)x},\quad t = \frac{a}{b}$$

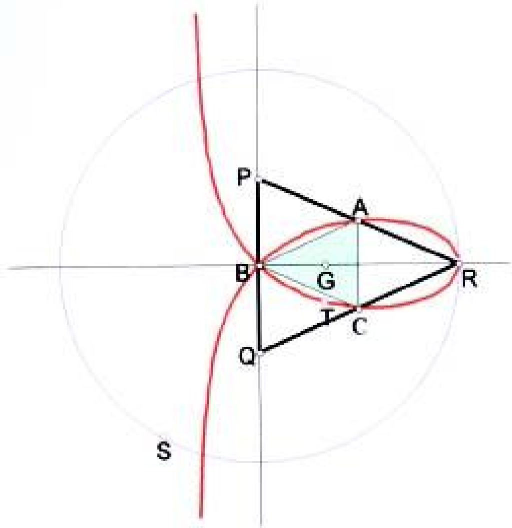
(1) $t = \sqrt{2},\quad y^2 = -\dfrac{1}{2a}x^3 + x^2$



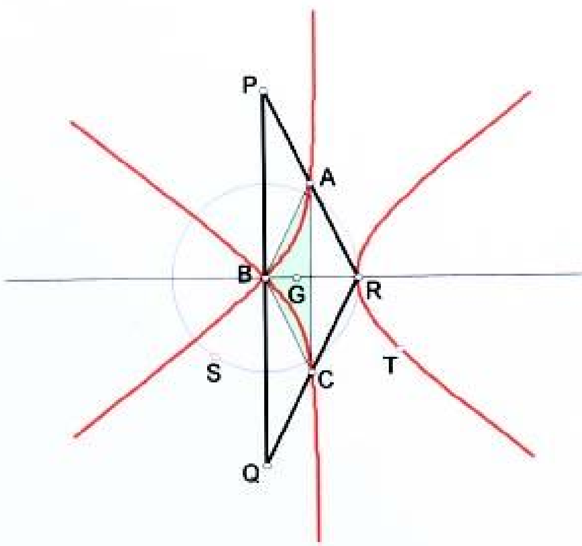
(2) $t = \sqrt{3}$, $\triangle$ PQR is equilateral. $y^2 = \dfrac{x^2(2a - x)}{2a + x}$ is the strophoid.



(3) $t=\sqrt{5}$, $y^2=\dfrac{x^2(2a-x)}{2a+3x}$ is the folium of Descartes



(4) $t=\dfrac{1}{2}$, $y^2=\dfrac{x^2(8a-4x)}{8a-7x}$



Theorem 7 Assume that $\triangle PQR$ is isosceles. Let G be the centroid. Let S be an arbitrary fixed point. If $\triangle PQR$ moves along a line rigidly, then the curves of C-transformation of S under G form a family of cissoid.

Proof Let P(0, b), Q(2,b-2t), R(2,b+2t), S(f, 0).

Then S(G)=T(x, y) with

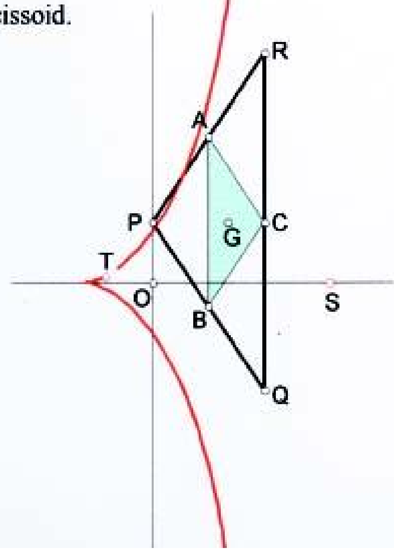
$$\begin{cases} x = \dfrac{2(-4t^2f + b^2 + t^2f^2)}{b^2 - 8t^2f + 3t^2f^2} \\ y = \dfrac{b(b^2 - 12t^2f + 3t^2f^2 + 8t^2)}{b^2 - 8t^2f + 3t^2f^2} \end{cases}$$

Then

$$y^2 = \frac{(3f-8)t^2}{f} \cdot \frac{x^3 + \frac{2(3f^2-15f+20)}{3f-8}x^2 + \frac{(-24f^2+68f-64+3f^3)}{3f-8}x + \frac{2(8f^2-20f+16-f^3)}{3f-8}}{2-x}$$

$$f = 2 + \frac{2}{3}\sqrt{3}, \quad t = \frac{1}{3}\sqrt{9+6\sqrt{3}}, \quad \frac{(3f-8)t^2}{f} = 1$$

$y^2 = \dfrac{(x+a)^3}{2-x}$ is the cissoid.



4.Summary 5.References 6.Software

SUMMARY

1. The Euler line is determined by the orthocenter and the conjugate Euler line is determined by the incenter under the C-transformation.
2. Conics are generated from straight lines under the C-transformation. The relative position of the straight line and the circumscribed ellipse determines whether the conics are circles or ellipses or parabolas or hyperbolas. A new synthetic construction with rulers of conics passing through five points is discovered.
3. The family of deltoid, including hypotrochoid and the extension of hypotrochoid with same nodes, are generated from circles with the centroid of the triangle as center under the C-transformation. This leads to a new construction of the generalized deltoid passing through five points.
4. The family of strophoid and the family of folium of Descartes are generated from circles with the midpoint of the base of the triangle as center under the C-transformation. In addition, these two families coincide under the C-transformation.
5. The family of cissoid are generated from a triangle moving along a line rigidly under the C-transformation.

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2. Chau Haur Guu, Dictionary of Mathematics, Jiannhorng Company
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SOFTWARE

1. Geometer's Sketchpad
2. Maple V

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