2021 年臺灣國際科學展覽會
優勝作品專輯

作品編號　160014

參展科別　物理與天文學

作品名稱　實驗室裡的飛行荷蘭人--複雜蜃景之探究

(Fata Morgana: the explanation of the 'Flying Dutchman')

得獎獎項　大會獎 二等獎

巴西MOSTRATEC正選代表

就讀學校　嘉義縣私立協同高級中學

指導教師　何世明、林俊元

作者姓名　林姵妤

關鍵詞　海市蜃樓、折射率梯度、全反射

作者簡介

我是來自嘉義縣協同高級中學的高二學生林姵妤。在從小參加各種科學研究活動的過程中，學習到科學不僅止於課本、論文或實驗室，而是充斥在日常生活甚至古代傳說中。在這次實驗中，利用生活中容易取得的材料和科學方法試圖對「飛行荷蘭人」的傳說做出科學性的探討，很開心能有機會能夠在 2021 國際科展的舞台上分享我的研究結果。

Abstract

Fata Morgana is a specific form of complex superior mirage, mainly formed in high latitude ocean area and also believed to be the credible explanation of sightings of the 'Flying Dutchman'. In order to obtain the optimal conditions for forming and observing Fata Morgana, two main parameters, the variation of the refractive index gradients and the observation height, are investigated systematically through experimental observations and numerical simulations. By carefully control the diffusion of high-concentration sugar solution, the environment for the formation of multiple-image mirage can be reconstructed in a 60-centimeter tank as the refractive index gradient obtained in sugar solutions is much larger than that appeared in the atmosphere formed by air temperature gradient. To reveal how lights propagate in a medium with complex refractive index profile, a green laser is shined into the tank and the light trajectory is recorded by a side camera. The formation of mirage is verified by observing the image of a model boat located at the opposite side of the tank. A ray tracing program is also developed in this study to help to find the optimal experimental conditions and support the experimental observations. Our simulation results show a good agreement with the experimental results indicating our ray propagation theory is valid and the optimal condition to form and observe Fata Morgana is obtained.

摘要

複雜蜃景(Fata Morgana)專指擁有多重影像的特殊上蜃景，形成於高緯度海面上，也是「飛行荷蘭人」的主要成因。本研究主要藉由探討介質折射率梯度變化與觀察者高度位置等變因，釐清複雜蜃景形成與觀察的最佳條件。藉由控制高濃度糖水溶液擴散形成的密度梯度，我們在六十公分的水缸中重建出形成複雜蜃景的環境，主要是因為糖水溶液中的折射率梯度遠大於海面上空氣的逆溫梯度所造成之折射率變化。為了進一步解析光在複雜折射率介質中之行進模式，我們以綠雷射光入射糖溶液，在側向以相機紀錄光的行進軌跡，分析探討其折射現象。我們同時利用相機觀察放置於水缸另一側的模型船，藉以觀察實際蜃景的形成與演化。本研究中我們另發展一套光軌跡的模擬程式，以協助實驗的進行與驗證實驗的成果。藉由實驗與理論模擬相互映證，充分探討複雜蜃景的成像與形成的最佳條件。

1. Introduction

1.1 Motivation

Mirage is a fascinating phenomenon produced by the bending of light rays in a non-uniform medium to display objects or sky at a far distance. The refraction of light occurs because large variations of the air temperature results in the existence of a refractive index gradient in the atmosphere. Researches about mirages with single image had been done over the decades in experiment [Greenler 1987, Barker 1989, Schricker 2001, Lopez-Arias 2009] and in numerical simulations [Trankle 1999], but there are very few researches about Fata Morgana [Young 2017], since it is believed to be the most complex type among other mirages. In order to demonstrate mirage in a Lab., Barker et.al. used heater to warm up the air in order to produce refractive index difference, which is very hard to control. Other researchers mixed alcohol, salt water or other high-refractive index solutions with water to produce non-uniform media but lack of the precise control of the refractive index profile. Since a complex non-monotonic refractive index gradient is required to produce Fata Morgana, fast mixture of various materials makes the refractive index profile unstable and the corresponding optical phenomenal becomes difficult to observe. In this study, we were motivated to investigate Fata Morgana in a different way, which use the diffusion between saturated sugar solution and water to create a complicated refractive index gradient and set the stage for the complex superior mirage. It is noted that only saturated solution can greatly reduce the diffusion time and make the evolved refractive index profile smooth and stable for the experimental observation as can be seen in the following chapter.

1.2 Research Purpose

(1) Investigate the formation of mirage with changing refractive index gradient.

(2) Investigate the formation of mirage with changing observation height.

(3) Provide a convincing theoretical model to predict the formation of mirage.

(4) Discuss the optimal condition to observe Fata Morgana.

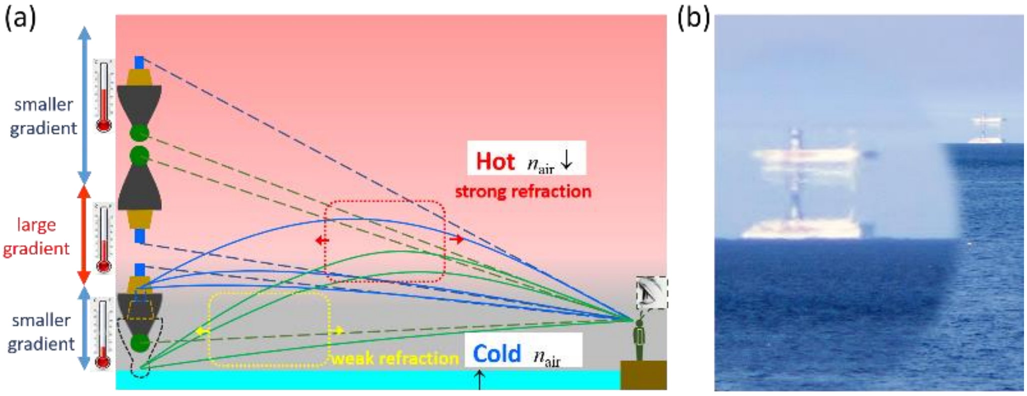


Figure 1.1 (a) Fata Morgana is formed by the refraction of light when it passes through a medium with non-monotonic refractive index gradient. (b) 4-ship mirage (www.atoptics.co.uk /fz782.htm).

1.3 The definition of Fata Morgana

In a very warm day, when the tide flows in and brought cold water into the strait, the air above the sea level is chilled. The atmosphere then has a temperature gradient with the upper part warmer than the lower part, resulting in a refractive index gradient with the upper part lower than the lower part as illustrated in Fig. 1.1(a).

The light ray from the object propagating through the cold and warm air layers is refracted downward at various degrees before entering the observer's eyes. Since the eyes tend to trace light back as straight lines, light rays pass through cold air near the sea level is hardly deviated. The image of the boat is formed just above its original position.

For light ray travels between the boundary of cold and warm air layers is refracted toward colder and denser air and thus forms an inverted mirage image. The large-angle light ray penetrating deeply into the cold and warm air boundary is refracted strongly due to a large refractive index gradient thus forms an up-lifted image of the boat.

As can be seen, Fata Morgana is a type of superior mirage with at least three or more upright and inverted images (Fig. 1.1). To demonstrate this unique phenomenon, a non-monotonic variation of temperature or air density profile along the height is necessary, namely, a relatively large refractive index gradient air layer sandwiched between two small refractive index gradients layers. This

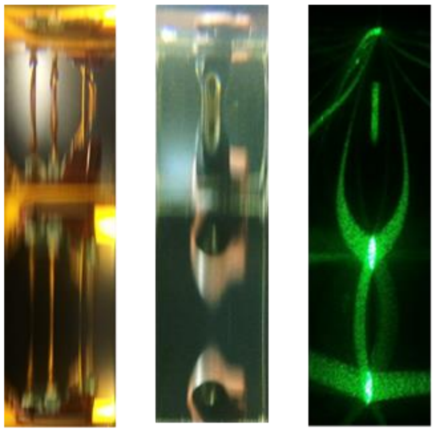


Figure 1.2 Observation of five-image Fata Morgana of model boat (left), laser pointer(middle) and laser light(right) recorded 96 hours after diffusion of sugar solution.

requirement will be verified later with ray tracing simulations.

1.4 Observation in laboratory

Instead of using a temperature-dependent air density profile to bend the light path, we carefully control the concentration profile of sugar solution to create refractive index gradient needed for forming multiple mirages. In Fig. 1.2, the three pictures on the left show a five-image fata morgana of a model boat, laser pointer and laser beam respectively. The model boat helps us to identify the inverted images more clearly, while the shape of laser beam tells us if the light from the object is well focused. A smaller laser spot corresponds to a sharper image.

2. Experimental Method

2.1 Experimental setup

To reproduce Fata Morgana in our lab., we prepare a medium with variable refractive index gradient by carefully controlling the temporal evolution of diffusions between water and saturated sugar solution.

As shown in Fig. 2.1, the laser lights emitted from a laser pointer at various incident angles and height are used to illustrate the light ray trajectories. The side image of laser beams propagating in the tank is recorded with a camera located in the normal direction of the tank (side shot). We also set

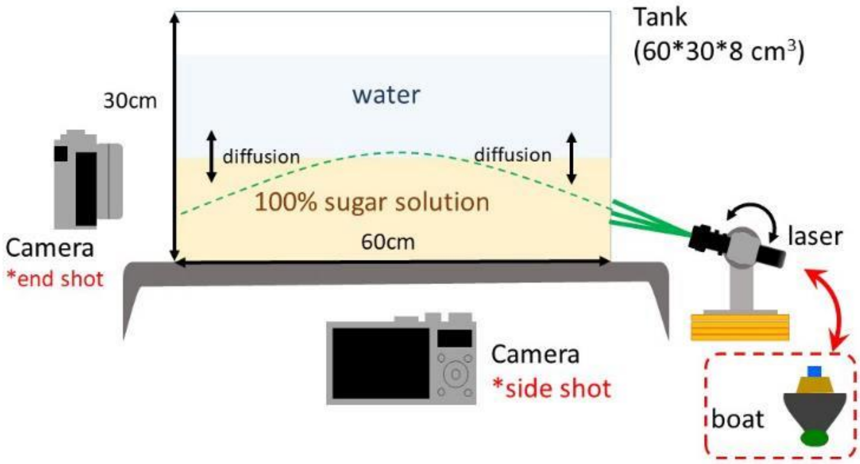


Figure 2.1 The schematic diagram of the experimental setup.

a camera on the opposite side of the laser pointer for recording the image of laser beams in the propagation direction (end shot).

When the laser pointer is replaced by a model boat, the end side camera can be served as an observer to record the multiple-image mirage of the boat within the field of view. Stacks are provided to vary the height of laser pointer or camera.

2.2 The measurement of refractive index

An efficient way of monitoring the refractive index of the sugar solution at different height can

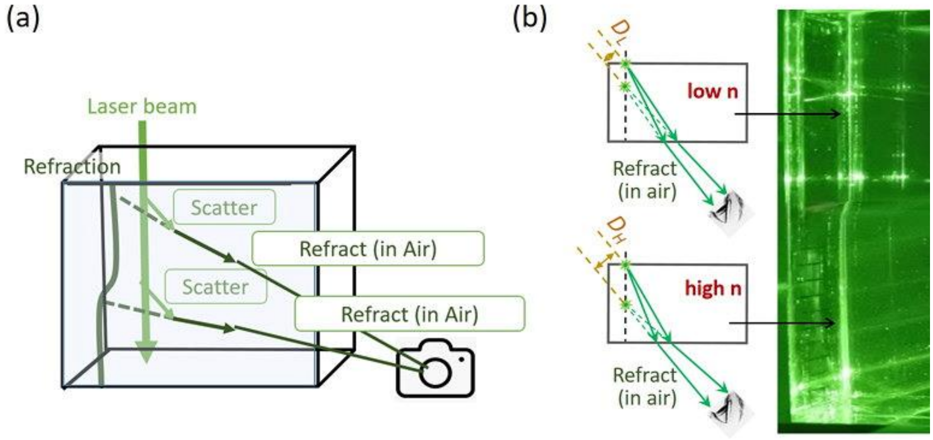


Figure 2.2 (a) Example of side shot image taken at 30 minutes after diffusion has taken place for refractive index measurement. (b) By shining a laser perpendicularly, the refraction of scatter can be used to derive the refractive index.

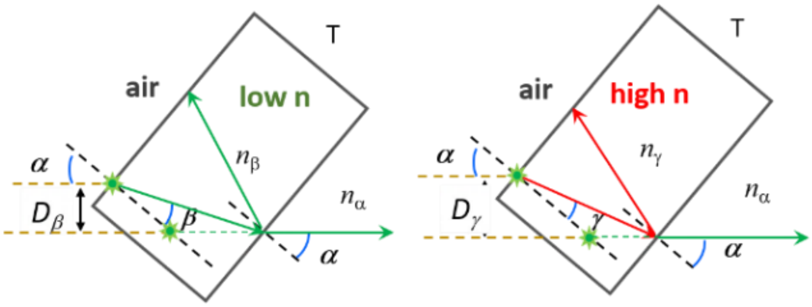


Figure 2.3 Light transmitting through different medium: light bends mores when it passes an interface with a larger refractive index difference. Stronger refraction results in larger horizontal displacement ($D_\beta < D_\gamma$).

be achieved by taking a side shot image of a vertical incident laser beam at a large angle.

Since the scattered laser beams have to transmit through sugar solution with different refractive index, the horizontal displacement of the laser beam can therefore be used to derive the solution's refractive index based on Snell's Law.

Assumed light passing through medium with refractive index n_β and n_γ , the incident angle as α, refracted angle as β and γ, causing replacement D_β and D_γ as shown in Fig. 2.3. The refracted index difference can be calculated with the following equation:

$$D_\gamma - D_\beta = \frac{T \sin 2\alpha}{2}\left(\frac{1}{\sqrt{n_\beta^2 - \sin^2\alpha}} - \frac{1}{\sqrt{n_\gamma^2 - \sin^2\alpha}}\right) \quad (2\text{-}1)$$

Derived using Snell's Law: $n_\beta \sin\beta = n_\gamma \sin\gamma$. We can found that the result of replacement difference is nearly proportional with refractive index difference. With this method, the refractive index profile during experiment can be fully recorded.

2.3 The observation of ray trajectory

A camera is utilized to record light trajectory from the side of the tank, which is the "side shot" (Fig. 2.4). In order to observe the trajectory of laser rays emitted at different angles, a diffraction grating lens is used to generate numerous diffracted laser beam at various angles. This can help us get strait view of the interlace of light trajectory, which is where Fata Morgana is formed. We changed the vertical position of the laser pointer by adding up stacks under the holder. Two centimeters are

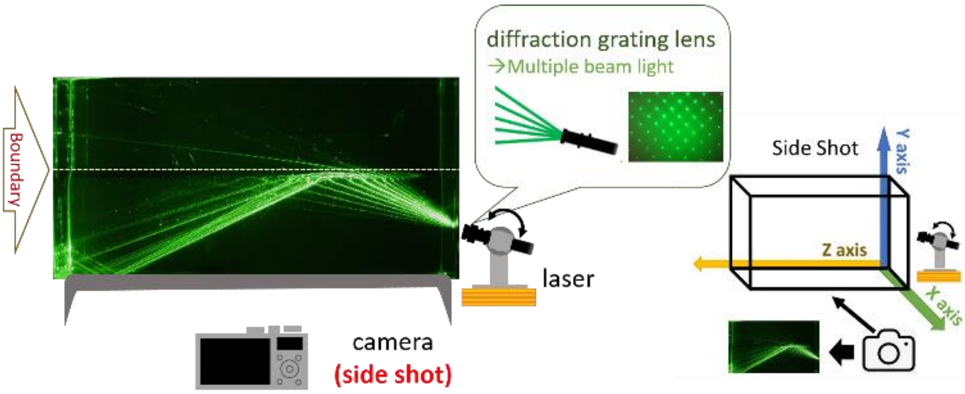


Figure 2.4 Example of "side shot" taken during experiment and the assigned coordinate axes.

added each time. Our Camera is fixed at the same position.

2.4 The observation of Fata Morgana

In Fig. 2.5, the camera of a mobile phone placed at the opposite direction of the object or the laser can take photos of the multiple image mirage, which is the "end shot".

The position of shots varies one centimeter each time. Calibration drown on the tank is used to adjusted the phone's position, making sure that the placement is accurate, since the deviation of a few millimeters will affect the result severely.

2.5 Experimental procedure:

(1) Produce refractive index gradient:

i. Put water into the tank, the ultimate height of water surface is 12.5cm.

ii. Put sugar solution into the tank with funnel. This can avoid bobbles and

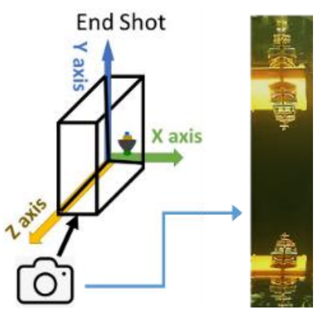


Figure 2.5 Schematic diagram of "end shot" for directly observing light transmitting through the test medium (diffused sugar solution).

unnecessary defusing in the fluid.

iii. Start timing the diffusing time. Refractive index profile is measured by taking extra side shot from large angle before each experiment.

(2) Turn off the light and take side shot of laser trajectory with single angle, which is 20 degrees.

(3) Take end shot of the laser light. The shots are taken at the various height, from the bottom to 24 centimeters above the bottom of the tank. The height is increased 1centimeter each shot.

(4) Take side shot of laser trajectory with multiple angles.

(5) Turn on the lights. Take end shot of the laser pointer. The shots are taken at the various height, from the bottom to 24 centimeters above the bottom of the tank. The height is added 1centimeter each shot.

(6) Replace laser pointer with model boat. Take end shot of the model boat. The shots are taken at the various height, from the bottom to 24 centimeters above the bottom of the tank. The height is added 1centimeter each shot.

(7) Replace model boat with laser pointer. The laser pointer is moved two centimeters higher every time by adding two stacks under. Repeat (2) ~ (6).

(8) Wait for a few hours for diffusion. Repeat (2) ~ (7).

3. Theoretical Model

3.1 Basic assumption

We assume the medium (air or sugar solution) along the horizontal plan is uniform and its refractive index is therefore constant in z axis but the refractive index profile is a complex function of height (y axis). These assumptions allowed Snell's Law to be applied as our foundation of theoretical model.

3.2 Theory

As illustrated in Fig. 3.1, for a ray at a grazing incident angle of θ translating a small distance delta z in the horizontal axis, the corresponding refraction angle and the vertical displacement delta y after passing through a boundary with a refractive index gradient can be calculated by using Snell's law:

$$n \cdot \sin(90-\theta) = (n+\Delta n)\sin(90-\theta-\Delta\theta) \qquad (3\text{-}1)$$

After some algebraic arrangements, we can find that a large deviation angle delta theta is obtained if a ray travels across a medium with a large refractive index gradient.

$$\Delta\theta = \frac{\Delta n}{n \cdot \tan\theta} \qquad (3\text{-}2)$$

and

$$\Delta z = \Delta y / \tan\theta \qquad (3.3)$$

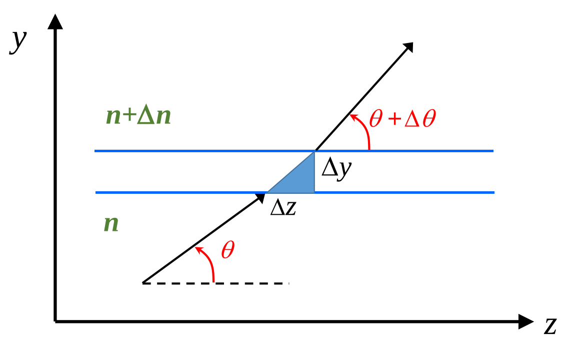


Figure 3.1 Refraction of light in a medium with a spatially varied refractive index profile. solution).

The same relation can also be obtained if we considering the Ray Equation in a differential form [Rabady 2013].

$$\frac{d^2 y}{dz^2} = \frac{1}{n(y)} \frac{dn(y)}{dy}$$

$$\Rightarrow \frac{d\theta}{dz} = \frac{1}{n} \frac{dn}{dy}$$

$$\Rightarrow d\theta = \frac{1}{n} \nabla n \cdot dz$$

Hence, a larger refractive index gradient can result in strong refraction. A larger deviation angle suggests the refracted ray curve towards the observer at a shorter horizontal displacement. Consequently, Fata Morgana is expected to be observed in a shorter distance.

3.3 Simulation

Using the theoretical model mentioned above, simulations are proposed to compare with experimental result.

3.3.1 Ray tracing simulation

We perform ray tracing simulation based on the theoretical model described earlier. With settled incident height, incident angle and y-axis-based refractive index profile, we were able to calculate the refraction angle and vertical displacement in a certain horizontal distance Δz. The information about the light trajectory in the first Δz can be used to derive the rest, demonstrating the complete light trajectory in the simulated environment. The above procedures are numerically calculated with Excel software. To further investigate the laser propagation in the diffused sugar solution, we use the measured refractive index profile in our raytracing simulation. We use red lines to represent light come from the bottom of the boat, blue lines to represent light come from the mast of the model boat (Fig. 3.2 & 3.3).

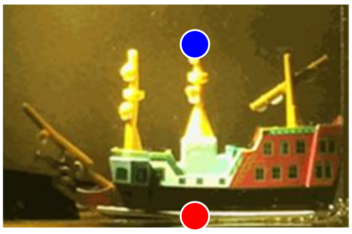


Figure 3.2 The photograph of model boat used in the experiment. The mast and the bottom of the boat are represented with blue and red dots respectively in simulations.

3.3.2 Simulated position of ray arriving at the observation plane (end side)

In order to find the optimal refractive index profile for producing Fata Morgana, we use the ray tracing simulation to predict the position of ray arriving at the observation plane. In the simulation graphs, blue circles represent the rays originating from the mast of the model boat, while the red circles represent the bottom of the boat.

3.3.3 Simulation of Mirage observed in different position

With the assistance of numerical simulation, we can obtain the location of each mirage as viewed with the end camera by tracing back the light rays' arriving angles at the position of ray arriving at the observation plane in straight line manner.

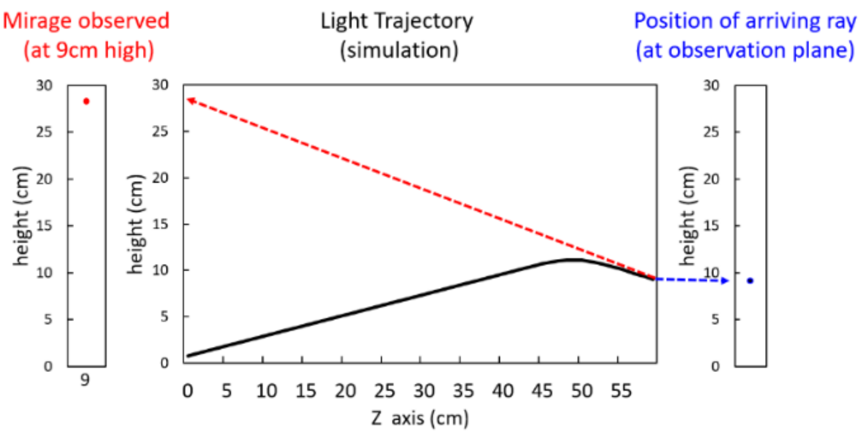


Figure 3.3 An typical example of a simulated ray trajectory in diffused sugar solution.

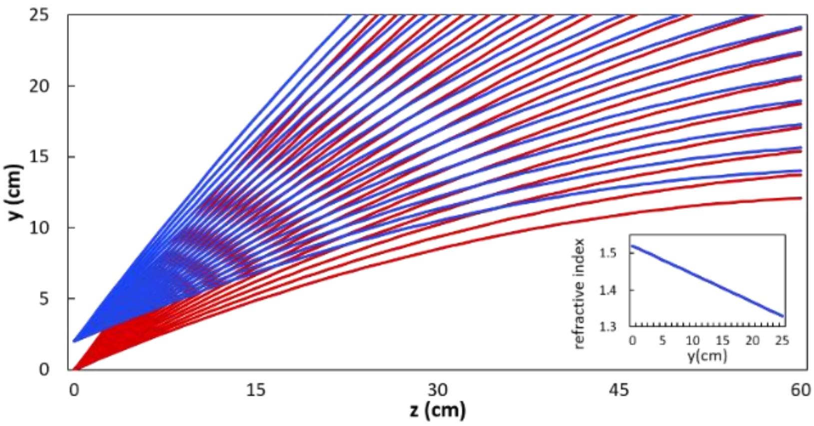


Figure 4.1 Simulated ray trajectory of laser beam at various incident in a medium with a linear refractive index profile (inset figure).

4. Result and Discussion

4.1 Simulated result of proper refractive index profile to observe Fata Morgana

Three different kinds of refractive index profiles are simulated and their light ray trajectories are plotted for comparison.

4.1.1 Linear index profile

The first case is a linear index profile with a negative constant gradient along the vertical axis y. In Fig. 4.1, the blue lines represent the rays from the upper part of the boat, and the rays from the bottom part are shown in red color.

According to ray tracing results, a linear refractive index profile can only form a single image of the boat at the same size. The only difference is the image position is up-shifted from its origin position. Multiple-image mirage (Fata morgana) does not happen in this case.

4.1.2 Exponential decay profile.

The second case is an exponential decay refractive index profile with a gradually decreasing

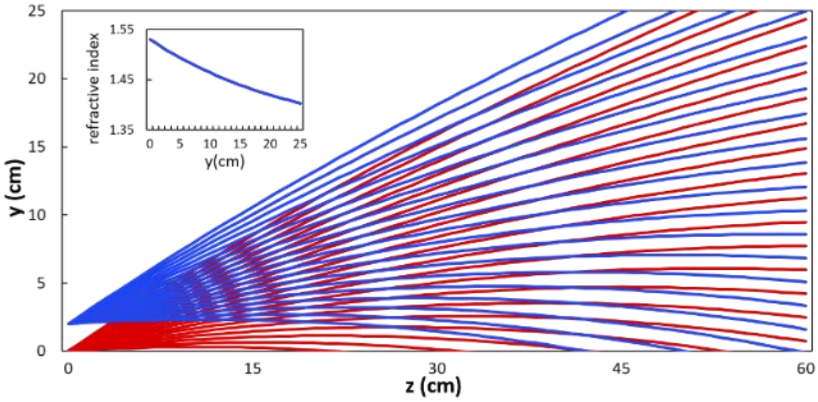


Figure 4.2 Simulated ray trajectory of laser beam at various incident in a medium with an exponential decay refractive index profile (inset figure).

gradient. The calculated ray trajectories in Fig 4.2 indicate that only one up-shifted image can be observed with its vertical size smaller than its original one. This condition does not form fata morgana either.

It's also noted that the change of the scale length in exponential decay profile does not vary the number of observation image. The size and position of the image are the only differences.

4.1.3 Error function profile.

The third one is a non-monotonic gradient distribution resulting from an error function refractive index profile which is a typical profile observed in concentration diffusions predicted by Fick's 2nd Law. In this type of medium, index gradient varies from small to large and to small value again.

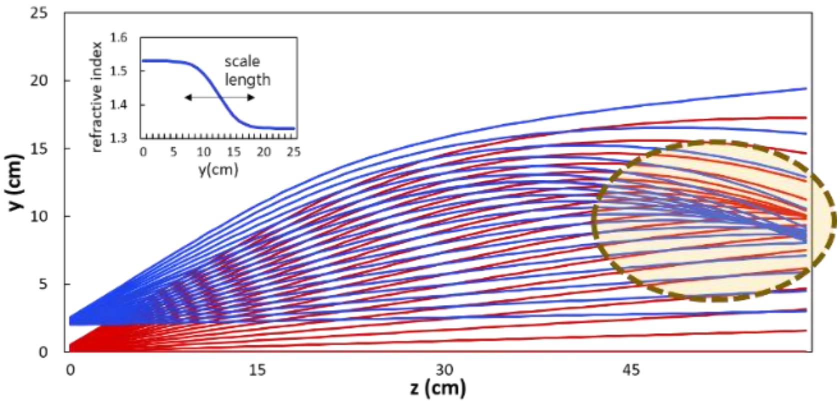


Figure 4.3 Simulated ray trajectory of laser beam at various incident in a medium with an error function refractive index profile (inset figure).

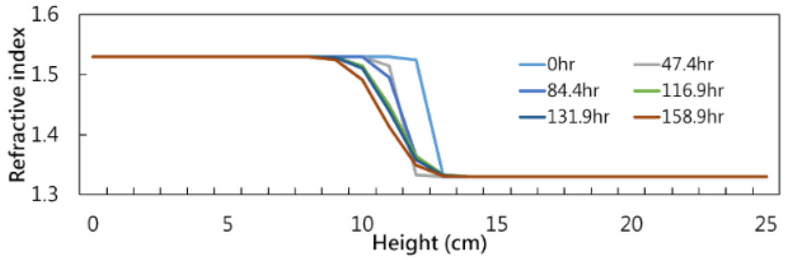


Figure 4.4 Temporal evolution of the measured refractive index profile with diffusion time.

As can be seen in Fig. 4.3, light rays originated from the boat of various initial angles start to interlace after they pass the area with complex refractive index gradient. For large-angle rays, they tend to experience strong refraction and bend toward the observer at large angle. On the contrary, small-angle rays are hardly deviated as the index gradient is small.

If an observer now locates at the region where light rays heavily interlaced, multiple-image at different viewing angles will then be observed. The image formed by crossed blue and read rays is inverted. Demonstration of Fata Morgana becomes possible.

4.2 Measured refractive index profile

The refractive index profile is measured with the method describes in section 2.2. It is found that the refractive index profile becomes flatter when the diffusion time increases as shown in Fig. 4.4. We use the scale length to define the width of the concentration diffusion region (from 92% to 8%). As the scale length increases with time, the refractive index gradient gradually decreases with time.

4.3 The affection of changing refractive index gradient

In the research, we vary the refractive index gradient by doing experiment at different diffusing hours. By taking side shot of light trajectory, end shot of observed image, and having theoretical simulations, the affection of changing refractive index gradient to the formation of mirage is believed to be fully investigated.

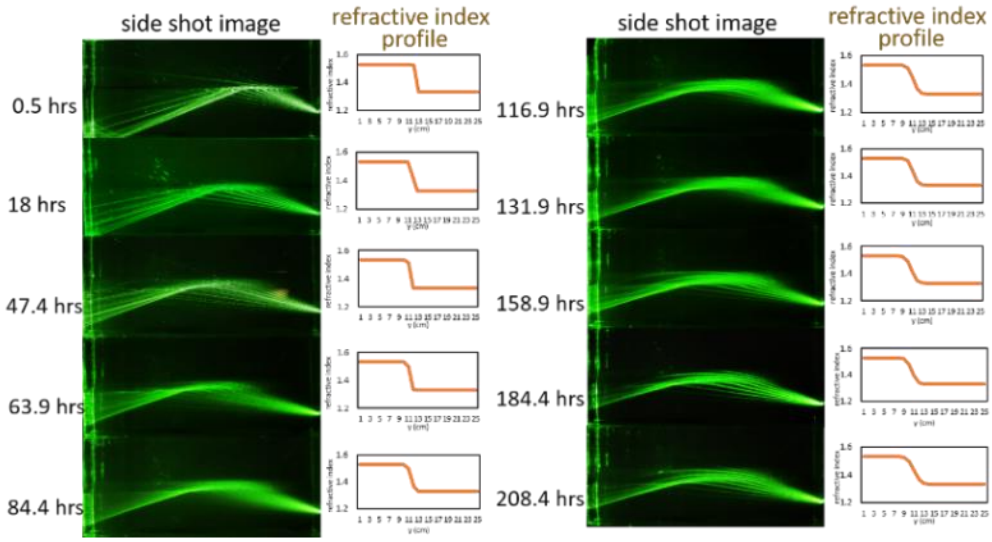


Figure 4.5 Ray trajectories under different refractive index gradients.

4.3.1 Ray trajectories under different refractive index gradients

In this section we show the ray trajectory recorded at different diffusion time. (Fig. 4.5) As the diffusion scale length in the refractive index profile increases with time, the index gradient becomes smaller and the refraction tends to be weaker. This makes the intersection of light rays move farther away from the object, the optimal distance for observing fata morgana is thus increased.

This experimental observation agrees with the prediction of our theoretical model: the smaller the refractive index gradient, the farther the distance to observe Fata Morgana.

4.3.2 Fata Morgana under different refractive index gradient

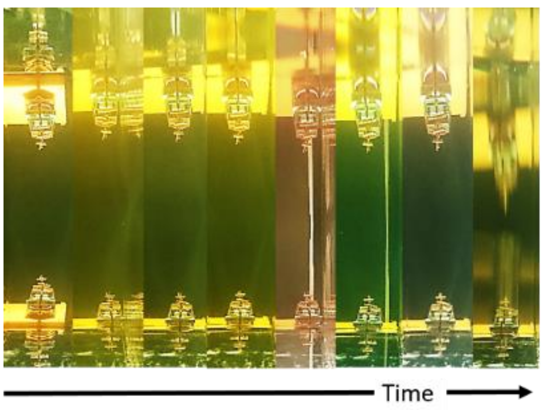


Figure 4.6 Fata Morgana under different refractive index gradients.

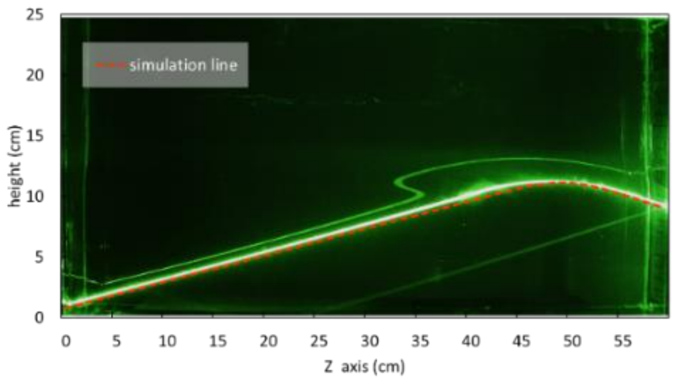


Figure 4.7. Side shot light trajectory compared with simulation result (red dashed line). The deviation between experimental result and simulation prediction is less than 0.1 cm.

To investigate the image of mirage, we replace the laser pointer with a model boat, and observe the image of the boat on the other side of the tank. In end shot images, at least 3 boat images (including original, inverted and up-lifted images) can be observed within camera's field of view (Fig.4.6).

As time passes, the refractive index gradient becomes smaller, the intersection of light rays moves toward the end camera, the number of mirage image which can be observed increases. This result indicates the diffused sugar solution can be an excellent medium for demonstrating Fata Morgana. (In this case from four to five.)

4.3.3 Simulation with measured refractive index

To further investigate the laser propagation in the diffused sugar solution, we compared our light trajectory with our raytracing simulation based on the measured refractive index profile. Our theoretical model agrees well with our experimental result (Fig.4.7). In fact, the deviation is smaller than 0.1cm, and most of the deviation was caused by manual operating tracking.

Confirming our simulation is accurate, we add in more rays with different incident angle. This simulation graph on the right shows the position of refracted light rays arriving at the observation plane. The blue circles represent the rays originating from the mast of the model boat, while the red circles represent the bottom of the boat (Fig.4.8).

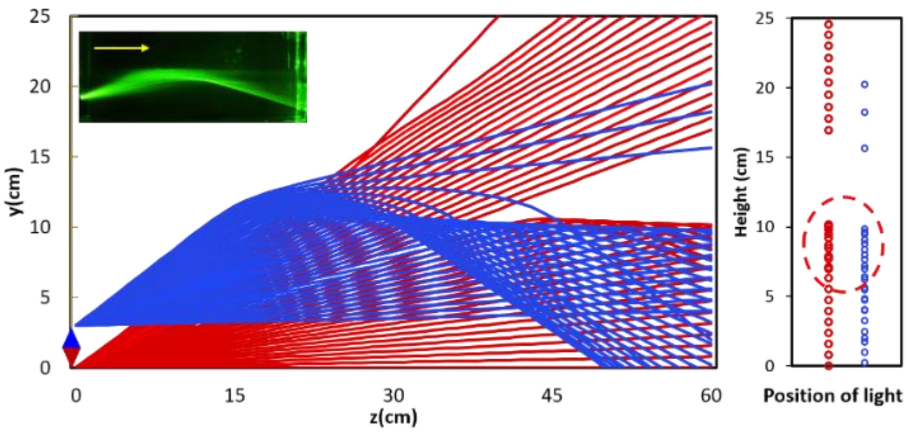


Figure 4.8 Ray tracing simulation with measured refractive index profile.

The overlap of several light spots at the height ranging from 6 to 10 cm at the observation plane also support the end camera's observation, indicating our simulation code can work correctly.

Then we make simulation of position of ray arriving at observation plane with measured refractive index at various diffusing hours. As the refractive index gradient decreases, the circles become more concentrated (Fig.4.9), implying a greater chance to see more images at the observing plane, agreeing with the experimental result that the intersection was moving towards the observation plane.

Although Fata Morgana may be seen at a certain range of observation position, the number of mirage images and their viewing angle can vary significantly with observer's height.

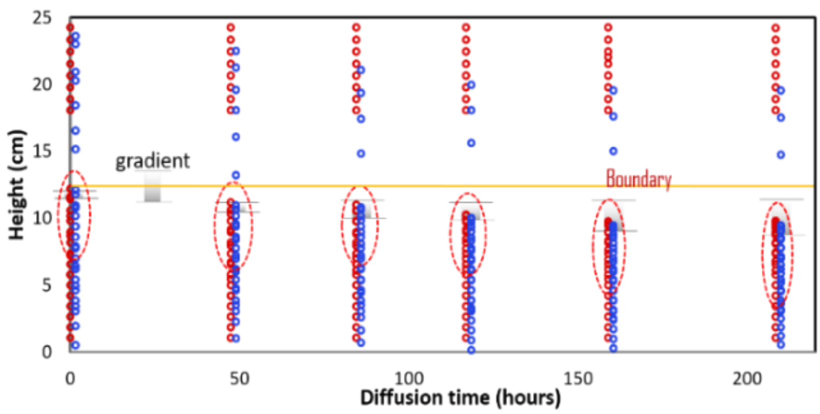


Figure 4.9 Ray tracing simulation with measured refractive index profile.

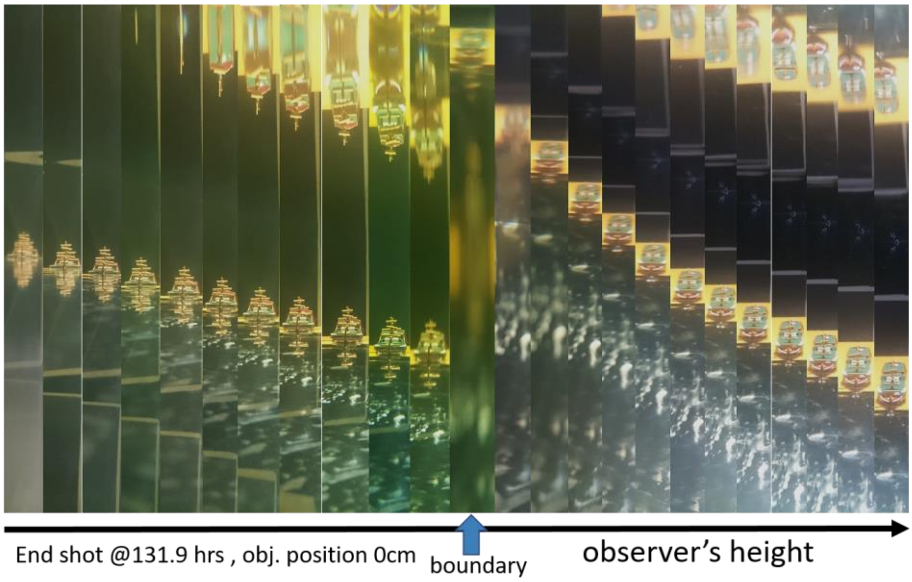


Figure 4.10 Variation of end shot image with the observer's height.

4.4 Formation of Mirage with changing observation height

Multiple images of mirage are formed because of the variety of angles when light gather at the same spot. Therefore, the position of the observer will significantly affect whether the mirage can be observed or not.

4.4.1 Observed Mirage in lab

These are the end shots taken at different heights at the observing plane. The image of the boat's mirage changes dramatically with observer's position (Fig.4.10). We can see that the best position to observe Fata Morgana is located at the region just below the diffusion zone centered at the original boundary.

4.4.2 Simulation

With the assistance of numerical simulation, we can obtain the location of each mirage as viewed with the end camera by tracing back the light rays' arriving angles at the observation plane in straight line manner.

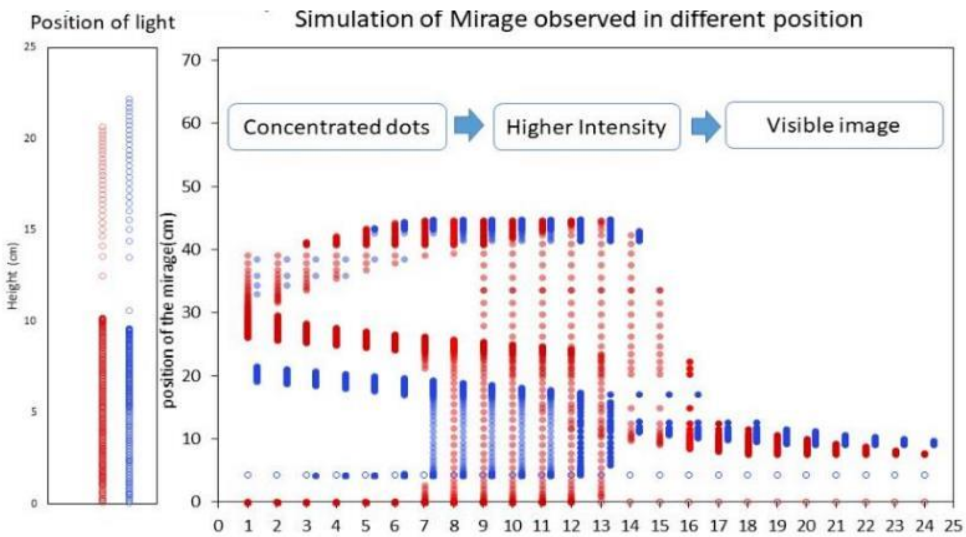


Figure 4.11 Simulation of (left) position of light arriving at the observation plane and (right) mirage observed in different observation height. Blue dots represent the mast and red dots represents the bottom of the boat, 300 different incident angles are considered for each light sources.

Simulation results show that fata morgana can only be observed just below the region with a large gradient (Fig.4.11). In this region, many light rays converge from various directions are the sources to form multiple images and thus produce Fata Morgana. Again, this simulation results agree well with our previous experimental observations.

To compare our theoretical model with taken image, we must adjust the simulation, since that as the shot is taken at outside of the tank, the refractive index of air and the tank should also be

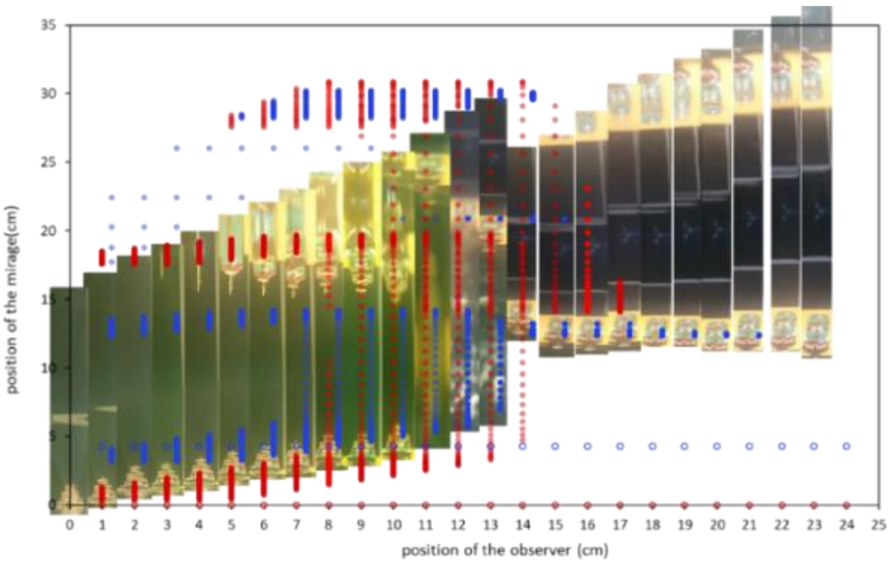


Figure 4.12 Simulation adjusted by considering the refractive index of the tank. The simulated images fit quite well.

considered. We didn't have them considered in the previous model, which actually better represents the Fata Morgana observed in the atmosphere, where Fata Morgana will not be observe behind a tank.

In this adjusted simulation you can see that the simulation fit quite well with our experimental result, and no matter observed with eyes or camera, the mirage will still appear when the observer is around the boundary between two mediums (Fig.4.12). Also, in the simulation spots representing mirage of parts of the model boat are made transparent and that can help us to clarify the light intensity of the image. As you can see that we can only observe image while the spots are concentrated. With unfocused light spot, the intensity will be too small for the image to be sharp and visible.

5. Conclusion

The optimal condition to observe Fata Morgana can be summarized as below:

1. **A larger sugar concentration gradient or larger air temperature gradient will result in a shorter horizontal observation distance**.

   Since the observing distance is linearly proportional to the scale length and the scale length is inversely proportional to the refractive index gradient, we can conclude that the optimal observation distance is also inversely proportional to the refractive index gradient (Fig.5.1).

   This also suggest if one would like to observed Fata Morgana in diffused sugar solution or in atmosphere, a larger sugar concentration gradient or larger air temperature gradient will result in a shorter observation distance (Fig.4.5).

2. **The observer's position also affects mirage formation in the field of view significantly.**

   Not only the number of mirage changes, the position of mirage also moves vertically. At different position, the intersection of light will have various combination of arriving angles, form a large variety of images.

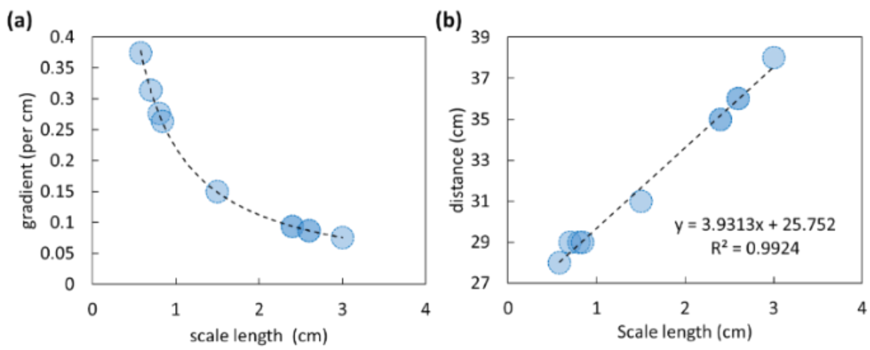


Figure 5.1. (a)Refractive index gradient is inversely proportional with scale length. (b)The best horizontal distance to observe Fata Morgana for different scale length of diffusion region.

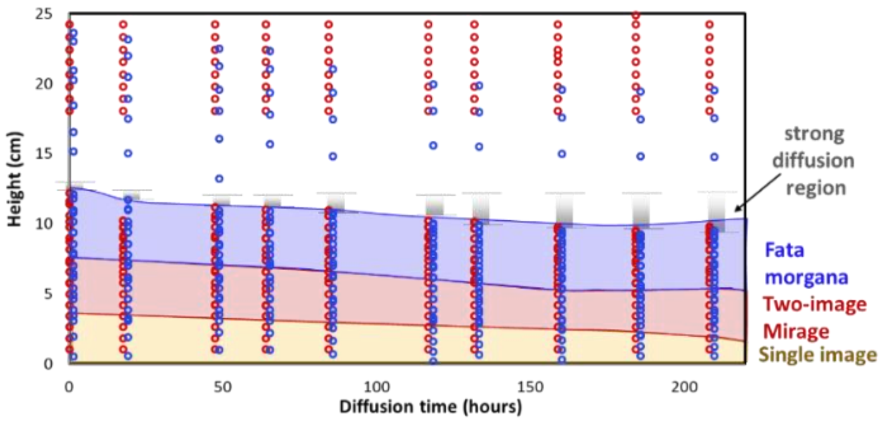


Figure 5.2. The simulation of best height to observe Fata Morgana, with different gradient (at various diffusion time).

3. Multiple-image mirage can be observed if an observer is located at a specific height and waits for diffusion taking place.

Both our experimental observations (Fig.4.10) and numerical simulations (Fig.4.11) show that it is most likely to observe fata morgana at the height just below the region where strong concentration or temperature diffusions in sugar solution or air occur and induce large refractive index gradient.

Our raytracing simulations predict that single (brown color), two-image (red color) or multiple-image mirage can be observed if an observer is located at a specific height and waits for diffusion taking place (Fig. 5.2.) .

Even the refractive index profile varies significantly with time, the optimal position to observed Fata Morgana is always located at a height just below strong diffusion region.

References

Barker, P. R., Crofts, P. R. M. and Gal M., 1989, A superior "superior" mirage, American Journal of Physics Vol.57, page 953.

Lopez-Arias, T., Calza G., Gratton, L. M. and Oss S., 2009, Mirages in a bottle, P HYSICS E DUCATION, Vol. 44, page 582.

Greenler, Robert G., 1987, Laboratory simulation of inferior and superior mirages, J. Opt. Soc. Am. A, Vol. 4, No. 3, page 589.

Rabady, Rabi Ibrahim, 2013, Simplified Model for Light Propagation in Graded-Index-Medium, Optics and Photonics Journal, Vol. 3, page47.

Schricker, Alexander, 2001, Refractive Bending of Light due to Thermal Gradients in Air, ATLAS Muon Note, Vol.17

Trankle, Eberhard, 1999, Simulation of inferior mirages observed at the Halligen Sea, Optics Express, Vol. 5, page 64.

Young, Andrew T., and Frappa, Eric, 2017, Mirages at Lake Geneva: the Fata Morgana, Applied Optics, Vol. 56, page G59.

Appendix

A-1. Light trajectory simulation with various wavelengths

The wavelength of light is proved to cause very little difference in this phenomenon. In the photos of Fata Morgana included in this report, it can be observed that the color distribution of the model boat mirage did not change in comparison with the original image of the model.

Our simulation also has the same result. We take the difference of refractive index caused by wavelength in to consideration and found only slight deviation.

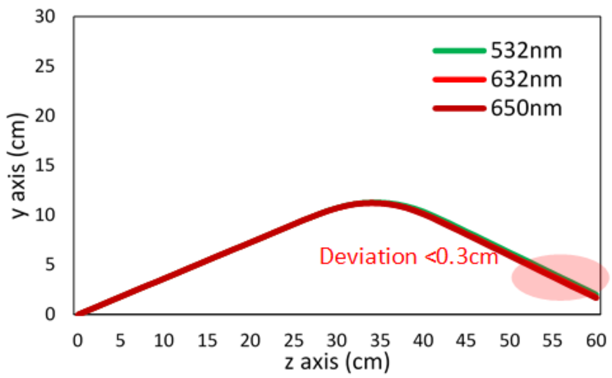


Figure A1. Simulation of light trajectory with different wavelength

A-2 Investigation on the affection of object's position

A-2.1 Light trajectory of light incident from different height

The change of object's position can be seen as the change of the position of large refractive index gradient. In the pictures of light trajectories incident in different height, it can be observed that as the

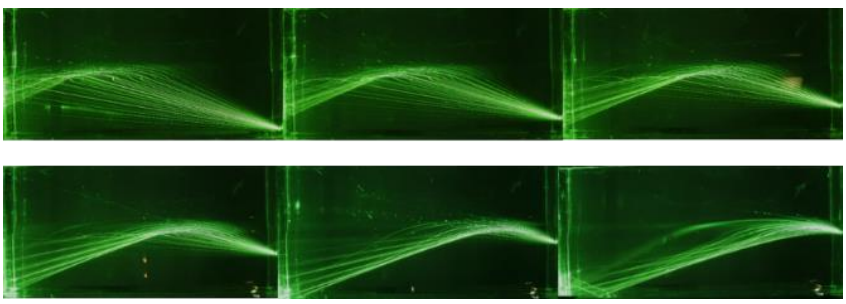


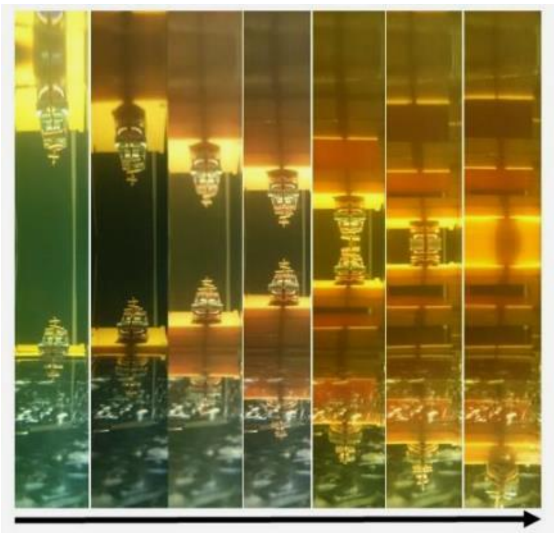
Figure A.2. Light trajectories of ray incident from different height.

position of laser moving higher towards the large refractive index gradient, the light propagates through larger refractive index difference and bend severely. The light incident in higher position experience the large refractive index earlier than those incident in low area. But it can be observed that the whole bending process can be seen as the same trajectory with horizontal displacement. Our simulation also agrees with our result.

A-2.2 End shot of object placed at different height

The end shots of model boat placed at various height show us clearly that the deformation and formation of multiple image will only happen to the objects that are under the area with large refractive index gradient.

Evident by light trajectory, we can see that when the object is placed at lower position, the total reflection of light in the large refractive index gradient will happen later and more distant from the object. That cause the adding chance to observe Fata Morgana since the light travel distant before it reaches the bottom of the tank. It's confirmed that the objects have to be under the region with large refractive index to produce multiple images.



Height

Figure A.3. Observed Mirage of object placed at different height.

【評語】160014

本研究藉由探討高濃度糖水溶液擴散形成的折射率梯度變化與觀察者高度位置等變因，釐清複雜蜃形成與觀察的最佳條件。並解析光在複雜折射率介質中之行進模式。本研究所發展之光軌跡的模擬程式，藉由實驗與理論模擬相互映證，是相當優秀的作品。