臺灣二ＯＯ八年國際科學展覽會

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作 品 名 稱：見微知「駐」-水珠律動與圓駐波

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學校 / 作者：國立新竹女子高級中學　　　　崔德昕

作者介紹

我是崔德昕，就讀於新竹女中資優班三年級。

國中時始參加科展競賽，獲得區域不錯的成績，並開啓了我對科學研究的興趣及寶貴經驗。高中時，毫不考慮的選擇了與日常生活息息相關的物理組。在一次專題研究課程中，老師播放了一段影片，而我深深被其中一個實驗吸引，並於課後主動跟老師討論此實驗的種種有趣現象，希望進一步了解，並希望能做更深入的探討。

經過一年的努力，有許多的挫折，實驗也重做了很多次，但藉由許多人的協助今日得以將我所觀察到的現象在此與大家交流分享，並擁有一般人所沒有的經驗，非常感謝那些曾經幫助過我的人。希望我能保有對這份研究的熱忱，迎向未來種種挑戰。

Dancing water droplets

It is always fascinating to see water droplet's dancing around on a Japanese teppanyaki hotplate surface. The water usually does not evaporate immediately, but form interesting shapes, dance around and follow by evaporation of gaseous water and disappear. In this experiment, we designed a very simple experimental set-up to observe the little water droplets dancing on a heated hotplate. A homemade stainless plate and a small heater attached to the plate, and a thermal couple was assembled. With this simple setup, we observed the variation of water droplet's shape as a function of the hotplate temperatures. The temperature of the water droplets, the duration of the water on the hotplate, and the shape number were measured. The shape formation mechanism was proposed. We found when the water droplet was subject to high heat due to the contact of the surface and the hotplate, the abrupt evaporation of the water molecules and violent vibration cause the formation of the various shapes to reach thermal equilibrium; the number of the shapes and the oscillation can be described by Laplace's equation, $r(\theta,\phi) = R + \sum c_n \cos(\varpi_n t) p_n(\cos\theta)$. Using a high-speed camera, we found the higher the temperature of the water, the more variations of the water droplet shapes can be observed. In addition, at a certain temperature range, the number of the water shapes did not change, suggesting a similar phase transformation behavior on the shape formation.

見微知「駐」—水珠律動與圓駐波

壹、研究摘要

緣起： 邂逅專題研究、水珠漫舞、剪輯影片，引起我們想更進一步揭開它的神秘面紗。
緣續： 了解熱平台上水滴大小的變化及水珠基本的形狀及變化律動。
緣繫： 進一步研究水珠多變的面貌，並探討水珠的大小、溫度、停滯時間及變化規律相關機制。
緣定： 糾纏在水珠圓舞曲中有如大珠小珠落玉盤的曼妙，其中埋藏了平均圓與能量量子化的律動。

貳、研究動機

偶爾和家人去吃鐵板燒時，看到廚師們在鐵板上加水或油時，板上的小水珠並沒有立即蒸發，而是不停的在熱鐵板上跳動滾動。隨後才以ㄘㄘㄘ的聲音快速蒸發消逝。因而想設計一套實驗，來探討水珠在熾熱板上微妙的變化情形及原因。

參、研究目的

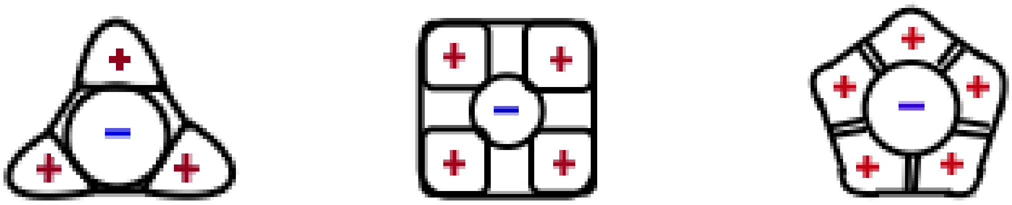
一、探討熱平台上水滴變化情形及水滴基本的變形形狀。
二、探討水滴在不同板溫下的不鏽鋼板上，水珠的大小、形狀、變化、溫度、停滯時間的相關性。
三、研究水珠形狀變化的規律及變形機制。
四、探討水珠在不同板溫下各種形狀出現的大小範圍及平均半徑關係。

肆、實驗原理

【觀念一】：振盪與水滴(water drop)變形

參考文獻【五】是一篇將水滴放置在 PTFE 或鐵氟龍板上，用喇叭(speaker)震盪水滴運動的實驗。從中得知，常溫下水珠的震盪和拉扯是由於本身是電偶極分子，當帶正電荷的水珠膨脹向上時，帶負電荷的水珠則向下陷落，反之亦然。

而由於多邊形水珠的巨大振幅，表面無法抵抗而破碎，導致水會從水珠中心噴射出去而造成變形。例如三角形：當三角形水珠變形時，正向和反向的三角形水珠交替著，表面激烈的波動一段時間就交換，水珠中心表面的垂直移動和三個角落表面的垂直移動是互補的。

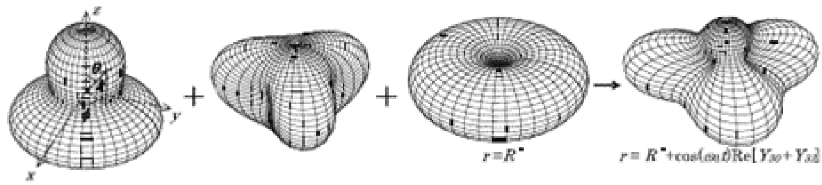


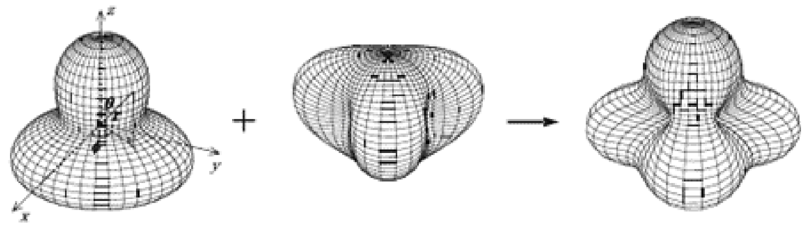
(a)Triangle　(b) Quadrilateral　(c) Pentagon

【觀念二】：

參考文獻【五】拉普拉斯(Laplace)曾對液體表面張力現象，以球形極座標(r,θ,ϕ)表示，在振盪的條件下，解出球形簡諧運動組合方程式

$$r(\theta,\phi) = R + \sum c_n \cos(\omega_n t) P_n(\cos\theta)$$



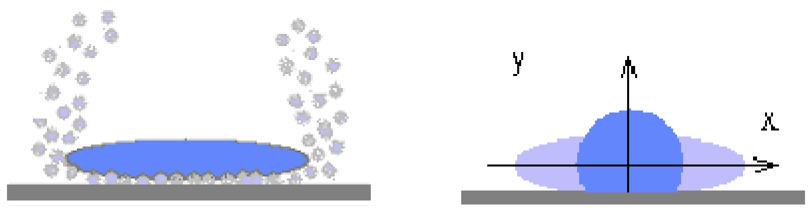


摘自：期刊 Exp Fluids(2006) 41:789-802

作者：Morihiro Okada．Minoru Okada 的研究報告

【觀念三】：蒸氣層與熱對流

高溫加熱板上的水滴於接觸瞬間，立即有水分子汽化成一層蒸氣膜，降低了 Hotplate 的熱傳導速率，水滴宛如一層蒸氣膜保護而形成穩定水珠，延長了停滯時間。如圖示：



水珠週遭空氣,在高溫加熱板上劇烈對流;加上高溫水珠內部分子的熱運動,形成水珠內外壓力差的作用,因此水珠不停的振動!轉動!變形!

【觀念四】：對稱變形與蒸散熱能

水珠處高溫狀態急待蒸散熱能，以降低溫度。高溫水分子的熱運動與空氣熱對流所造成的壓力差，致使水珠向外突出變形，以增大表面積加快蒸發速率，可以降低熱能。但是水珠變形的同時與 Hotplate 接觸表面積變大，單位時間熱傳導能量增加。表面張力內縮作用,水珠在突出變形與表面積縮小之下,劇烈變形後恢復球形,如此交替循環變化，如簡諧振盪一般。

伍、實驗設計

一、研究設備及器材

(1) 加熱器 Stirrer Hotplate(Corning) 平台
(2) K/J 型式熱電偶溫度計、兩支細針型
(3) 不鏽鋼板兩片 (19.7cm×16.3cm×0.1 cm)
(4) 錐形瓶 250ml、玻璃滴管
(5) 高速攝錄影機一台。Fujinon corporation 1:1.4/16mm HF16HA-1B
(6) 自行設計之鐵架（用以固定溫度計）。
(7) 16K 方眼紙(1mm)　193×266mm

二、實驗裝置

由左起為 K/J 型式熱電偶溫度計、加熱器 Stirrer Hotplate(Corning) 平台、錐形瓶 250ml 及玻璃滴管，正後方為自行設計之鐵架。

（圖一）

三、名詞定義：

（1）單位體積：滴管每滴落一滴之平均體積。
（2）停滯時間 T_m：水滴在加熱板上蒸發消逝的時間亦可稱為蒸發速率。
（3）圓形水珠：水滴在加熱板上形成扁球形之狀態稱為圓形水珠。
（4）邊形數：水珠變形時觀察者（含視覺暫留）所看到的邊形角數量。
（5）重影：水珠快速拉扯所造成視覺暫留之現象下所看到疊合的圖形。
（6）潛相熱：溫度升高提供熱源但邊形數沒變時，我們稱之為潛相熱。
（7）平均圓半徑：多邊形之內切圓與外接圓半徑的平均值。
（8）半徑限定：水珠平均圓半徑分布不連續現象。即某一邊形數在特定溫度，有特定的半徑範圍。

陸、實驗過程

實驗一：基本測量

〈表一〉 【a】水滴平均體積：

滴數	50	100	150	200	平均體積(cc)	單位體積(cc)
體積(cc)	1.98	4.02	6.00	7.98	0.0399	0.0400
體積(cc)	2.00	4.05	6.02	8.02	0.0401	
體積(cc)	1.96	4.00	5.98	8.00	0.0400	

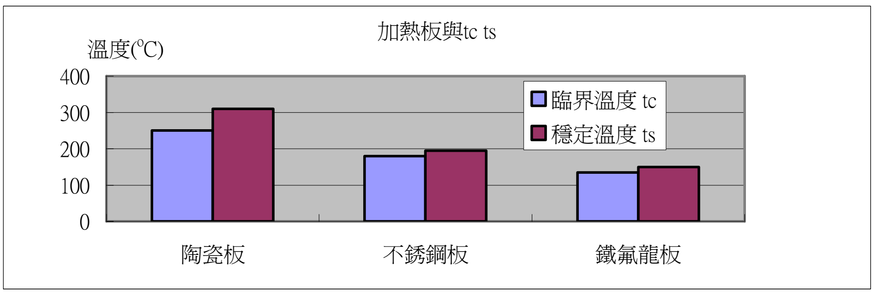
〈表二〉 【b】水滴大小（截面直徑）：不銹鋼板上

滴數	一滴	二滴	三滴	四滴	五滴
平均直徑(mm)	6.6	8.3	9.8	10.7	11.2

〈表三〉

Hotplate / 名稱	陶瓷板	平均溫度	不銹鋼板	平均溫度	鐵氟龍板	平均溫度
臨界溫度	248℃	250℃	180℃	180℃	138℃	135℃
	252℃		176℃		134℃	
	250℃		180℃		135℃	
	248℃		178℃		135℃	
	250℃		180℃		136℃	
穩定溫度	312℃	310℃	198℃	195℃	148℃	150℃
	305℃		196℃		150℃	
	310℃		195℃		150℃	
	308℃		192℃		148℃	
	310℃		195℃		152℃	

【數據分析】



（圖二）加熱板與臨界溫度和穩定溫度

《引自參考文獻〈三〉水珠漫舞》

註：

臨界溫度：水滴在加熱板上開始形成水珠的最低溫度

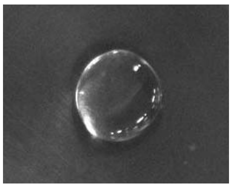
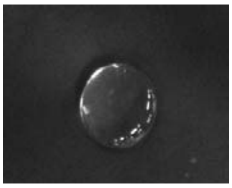
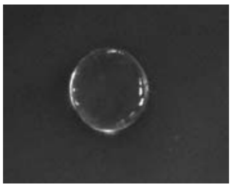
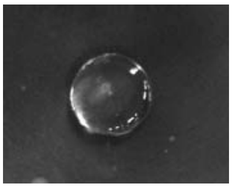
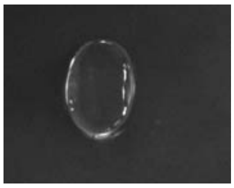
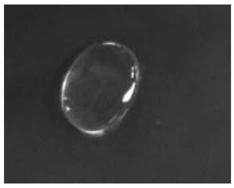
穩定溫度：水滴在加熱板上形成與滴入水滴體積相同時的最低溫度

【觀察結果】:

在（圖二）中我們發現，在不銹鋼板和鐵氟龍板上，臨界溫度較陶瓷板低，但由於鐵氟龍板的表面較粗糙，易對水珠的變形產生阻礙，因此我們決定選用不銹鋼板來進行以下實驗。

步驟一：紀錄及觀察各種形狀並給予定義

（表四）　　　　　　　　　　　　　　　　　　　　【】爲數據表中表示此圖形之代號

變形數	圓形【1】		十字形【4】	
肉眼所見				
高速攝影下				
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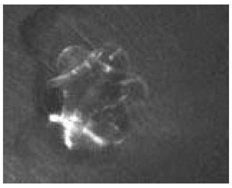
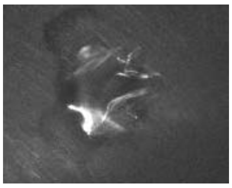
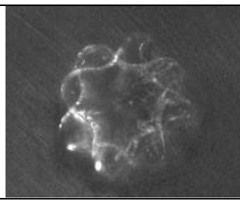
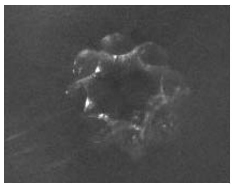
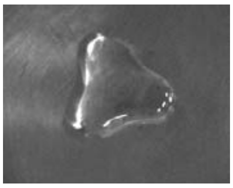
肉眼所見				
高速攝影下				
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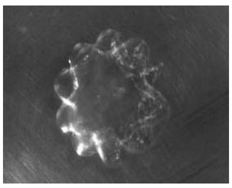
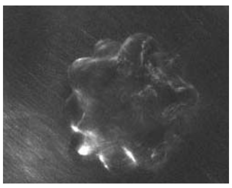
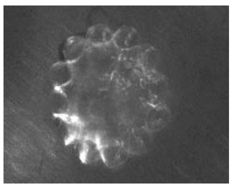
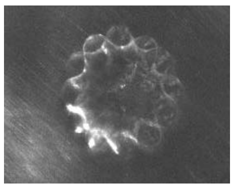
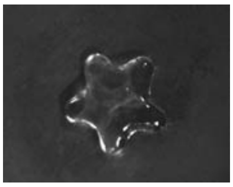
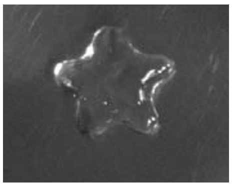
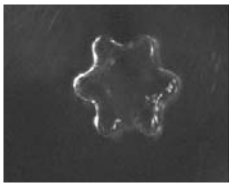
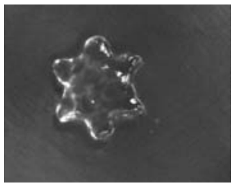
肉眼所見				

高速攝影下				
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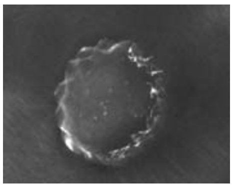
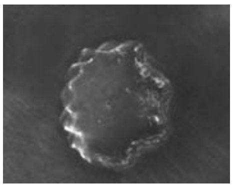
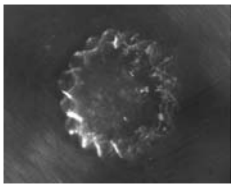
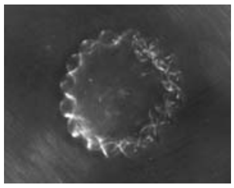
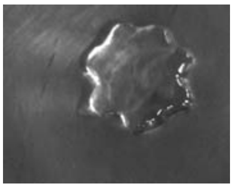
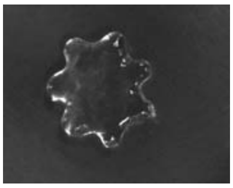
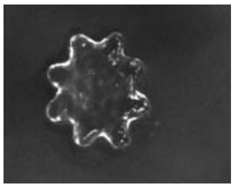
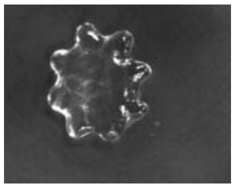
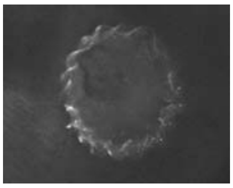
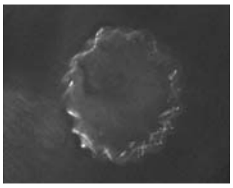
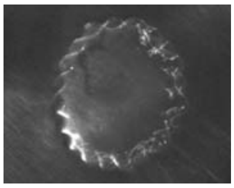
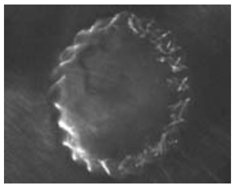
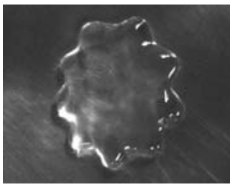
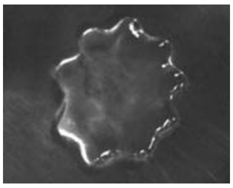
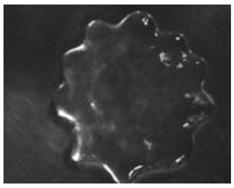
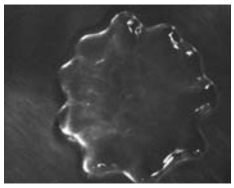
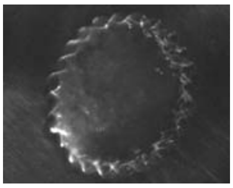
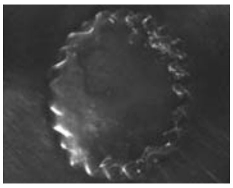
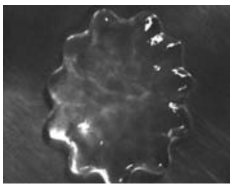
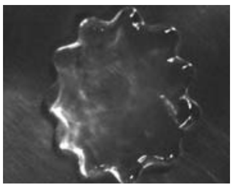
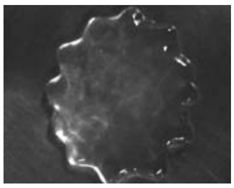
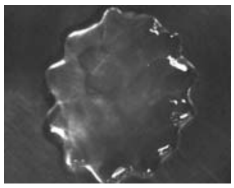
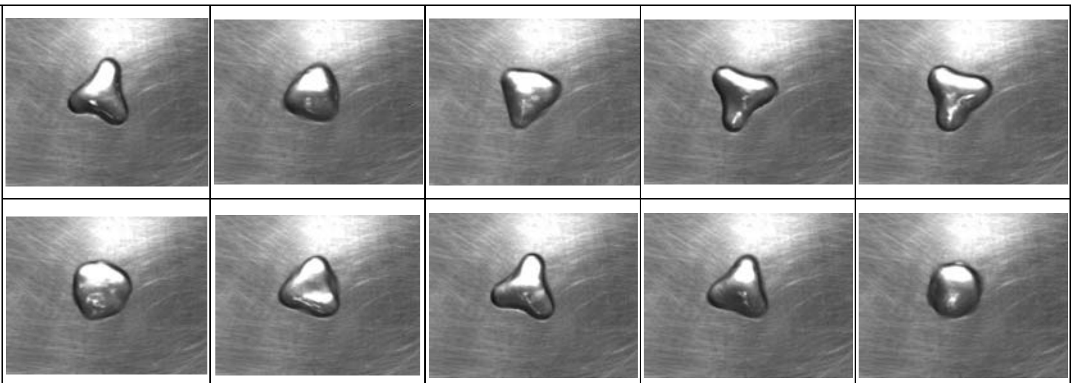
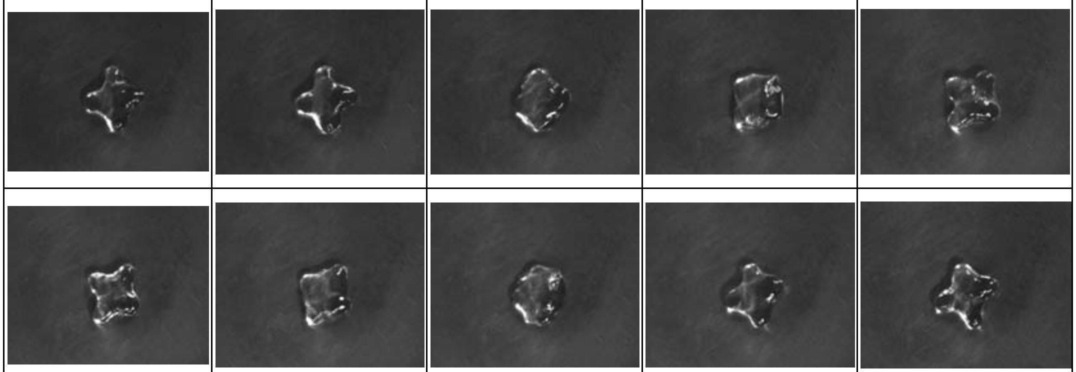
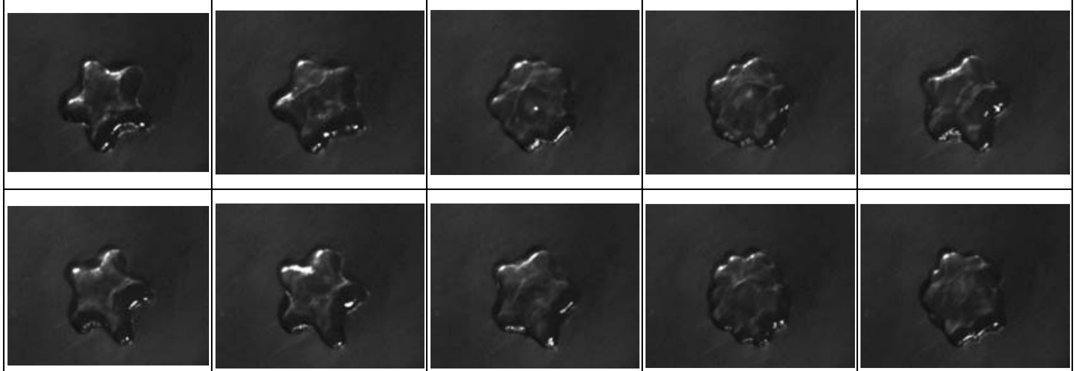
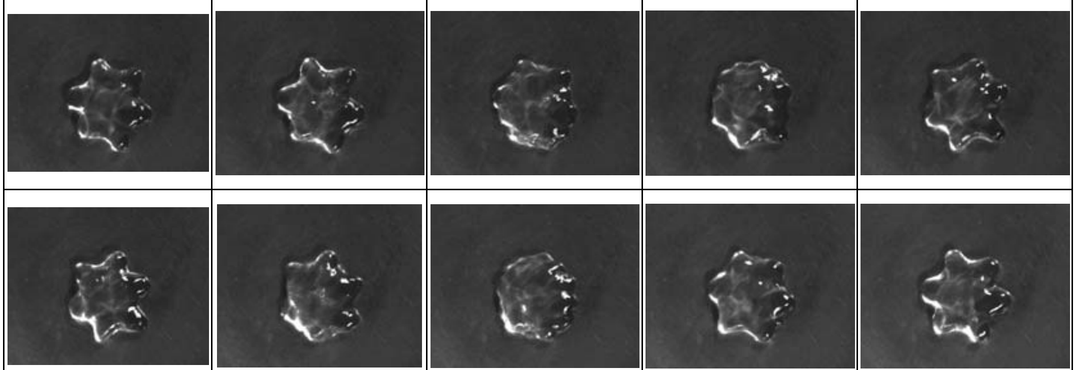
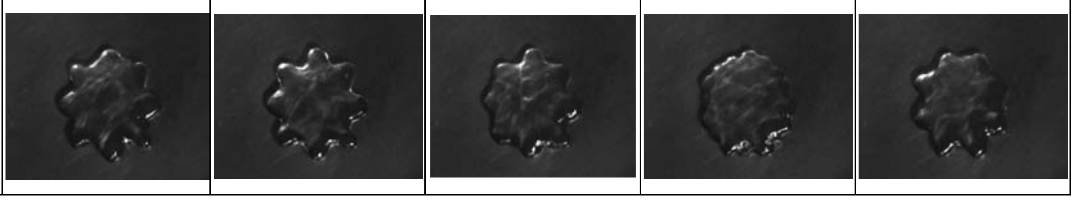
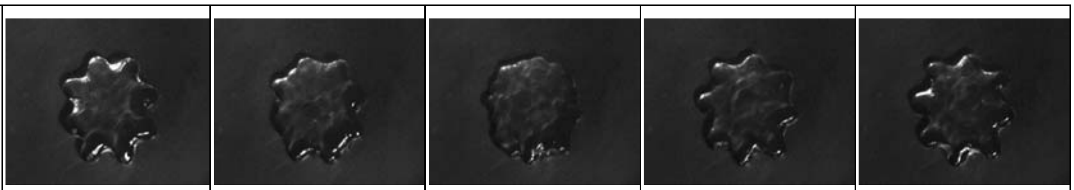
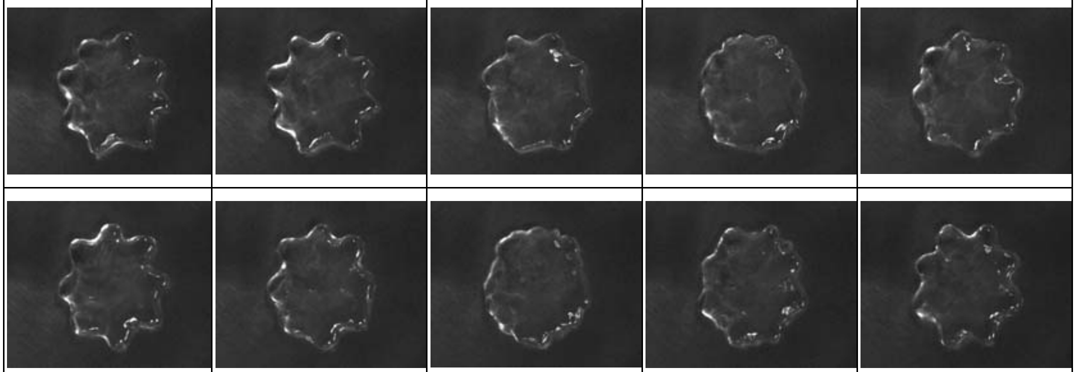
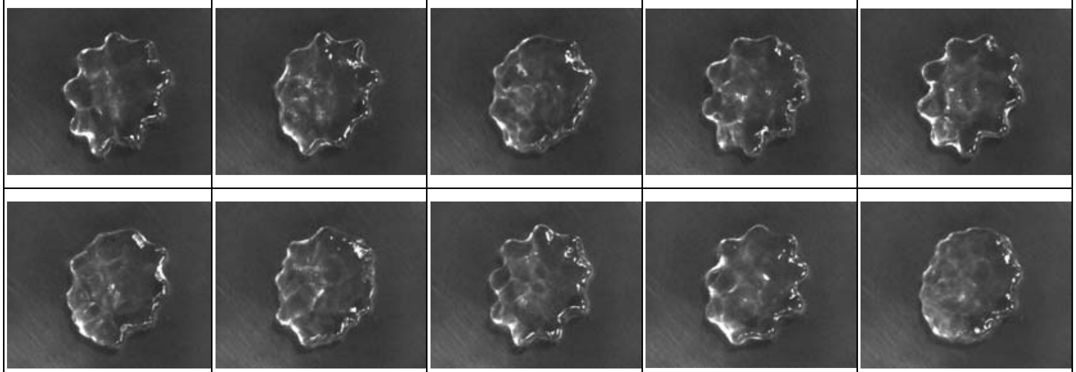
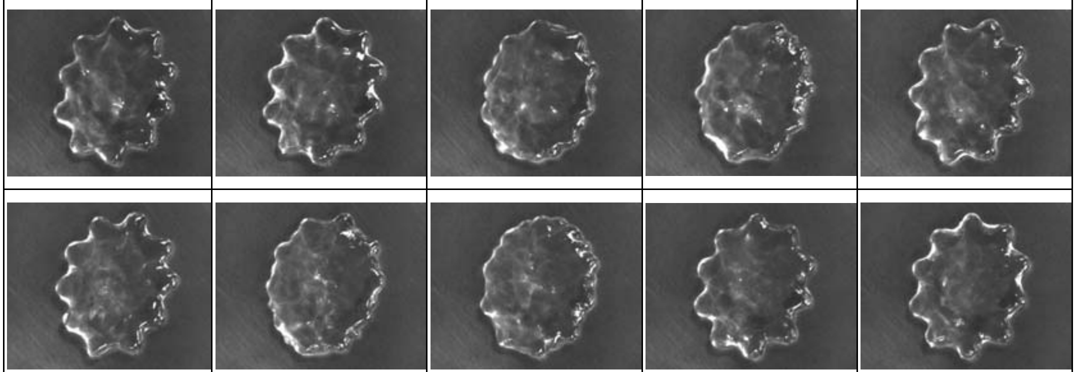
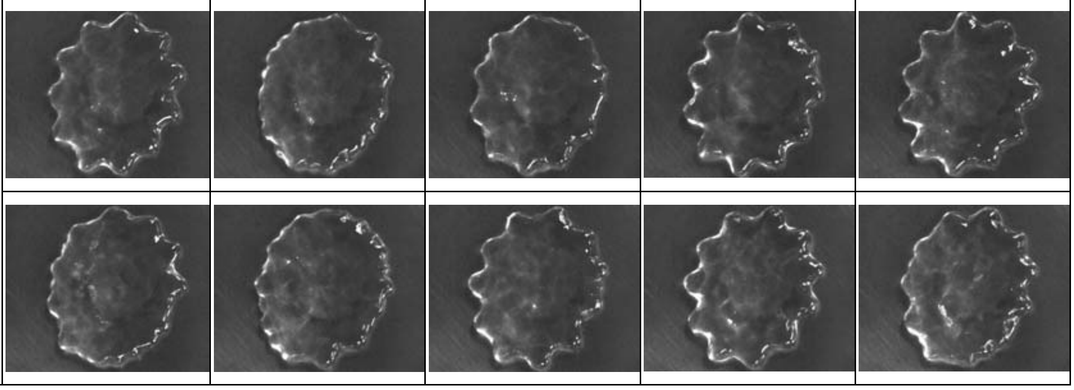
肉眼所見				
高速攝影下				
變形數	九邊形重影【18】		十邊形重影【20】	
肉眼所見				
高速攝影下				
變形數	十一邊形重影【22】		十二邊形重影【24】	
肉眼所見				
高速攝影下				

步驟二：用高速攝影機拍攝下所有圖形之變化規律

（表五）

三邊形	
四邊形	
五邊形	
六邊形	
七邊形	

	
八邊形	
九邊形	
十邊形	
十一邊形	

【實驗分析】：

於〈表五〉中觀察到水珠會在突出變形與表面積縮小之下，劇烈變形後恢復球形。如此交替循環變化且對稱變形，就如簡諧振盪一般。

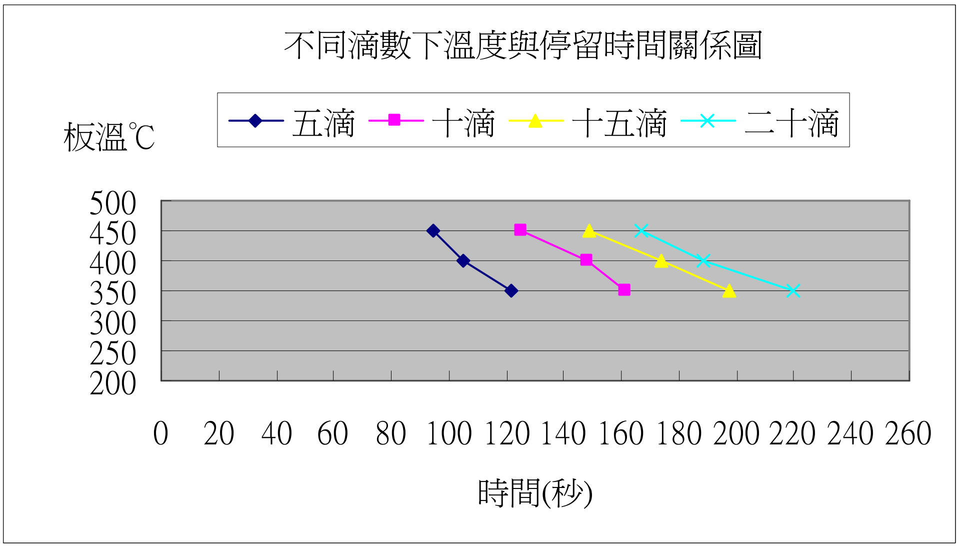
實驗二：水珠停滯時間在不同加熱板溫度下之探討

步驟一：將板溫分別依次穩定地控制在 350℃、400℃、450℃

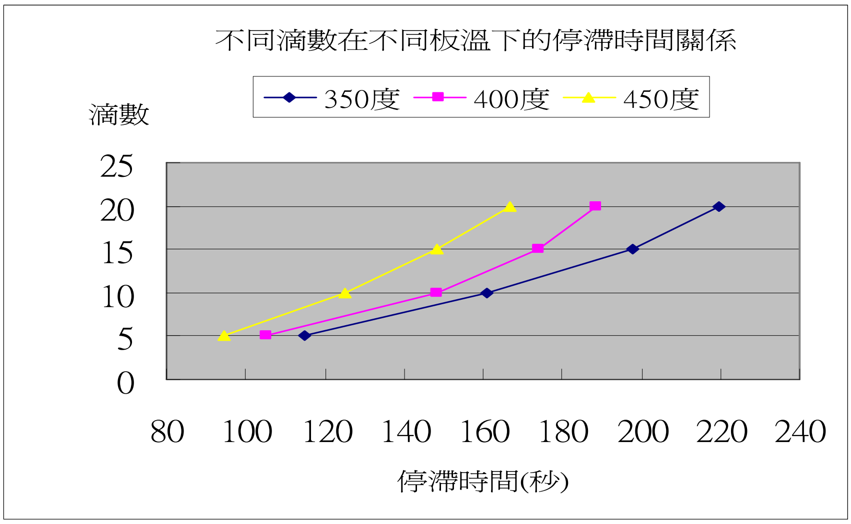
步驟二：於各個板溫下依序滴入五滴、十滴、十五滴、二十滴的水珠，並記錄下不同大小之水珠在不同板溫下的停滯時間，並畫函數圖以表現其關係。

（表六）

	350℃	400℃	450℃
5 滴	119.43	115.49	96.05
	119.28	98.74	95.59
	126.78	101.54	94.25
平均時間(秒)	121.83	105.26	95.30
10 滴	158.88	143.95	123.36
	163.70	149.10	121.78
	160.24	151.30	126.37
平均時間(秒)	160.94	148.12	123.84
15 滴	194.83	178.78	149.68
	193.91	169.29	148.26
	204.12	173.38	147.35
平均時間(秒)	197.62	173.82	148.43
20 滴	227.17	192.09	161.56
	213.32	186.12	162.47
	217.93	187.41	165.74
平均時間(秒)	219.47	188.54	163.26



〈圖三〉不同滴數下溫度與停留時間關係圖



〈圖四〉不同滴數在不同板溫下的停留時間關係圖

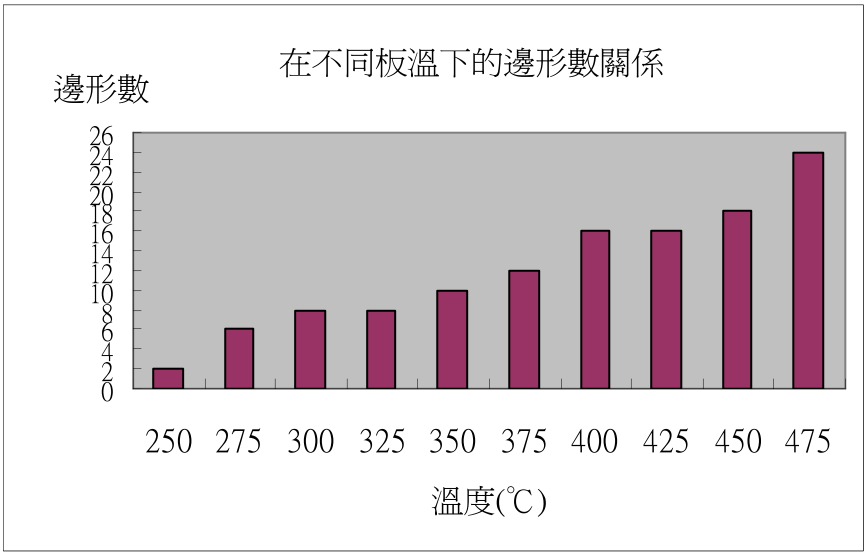
【實驗分析】:

於〈圖三〉中，我們觀察到水珠的停滯時間會隨著板溫的升高而變短。〈圖四〉中可發現水珠除了隨著水滴數之增加而停滯時間延長外，也會隨著水滴數之增加，在相同板溫下兩兩滴數之間(如五滴跟十滴)的停滯時間差值越小。從〈圖三〉中更可清楚的觀察出水滴數與板溫之停留時間之關係。

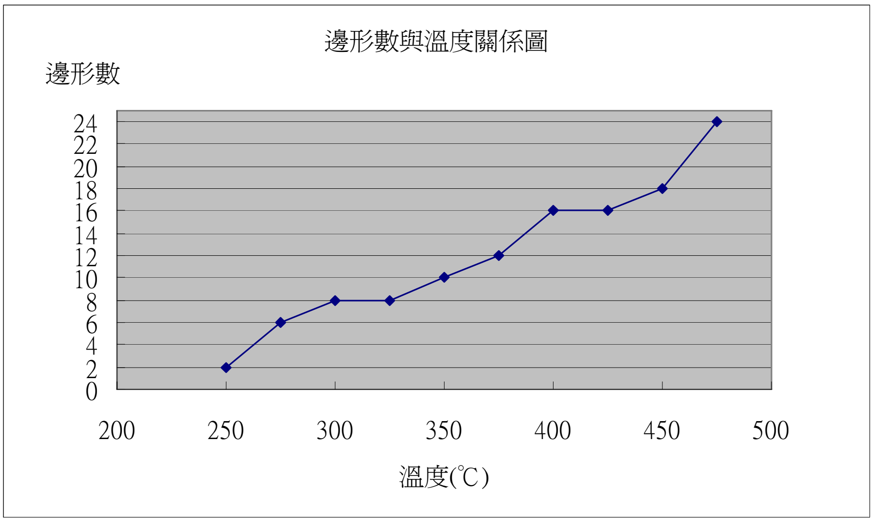
實驗三：水珠於不同加熱板溫度下之變形數關係

步驟一：將板溫分別穩定地控制在 250℃、275℃、300℃、325℃、350℃、375℃、400℃、425℃、450℃

步驟二：作數次的實驗並記錄所觀察到的圖形



（圖五）在不同板溫下的邊形數關係



〈圖六〉邊形數與溫度關係圖

【實驗分析】：

於〈圖五〉中明顯的顯示出：隨著溫度的升高，邊形數也隨之增大。〈圖六〉我們還觀察到 8 邊形(即四邊形重影)跟 16 邊形(即八邊形重影)為水平線，也就是說出現８邊跟１６邊形時，會有一段溫度範圍下都不會再增加邊形數，但只要溫度再持續升高到超過某一溫度，則又會出現更多邊形。

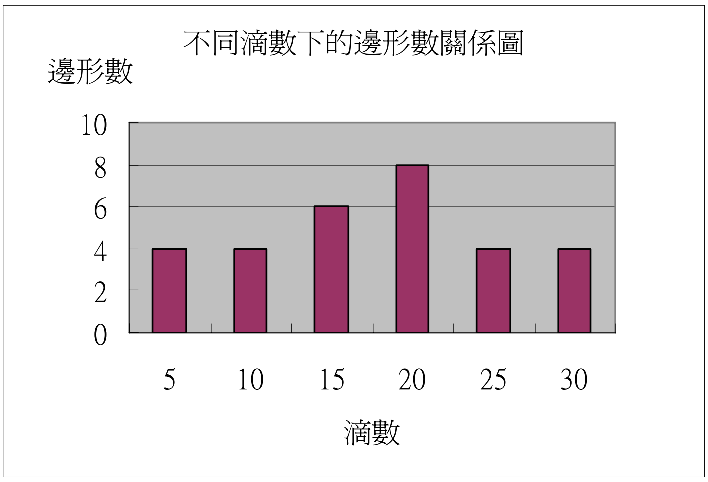
實驗四：水滴大小(滴數)與變形數之關係

步驟一：將板溫控制在 350℃，並依次滴入 5、10、15、20、25、30、35 滴水珠數次，並紀錄所出現的形狀。

〈表七〉 (＊表示為有出現之形狀)

邊形數 / 滴數	圓形 (1)	橢圓形 (2)	十字形 (4)	三邊形 (6)	四邊形 (8)
5 滴	＊		＊		
10 滴	＊		＊		
15 滴	＊		＊	＊	
20 滴	＊		＊	＊	＊
25 滴	＊	＊	＊		
30 滴	＊	＊	＊		

《各種形狀括號內的數字代表下圖中的邊形數》



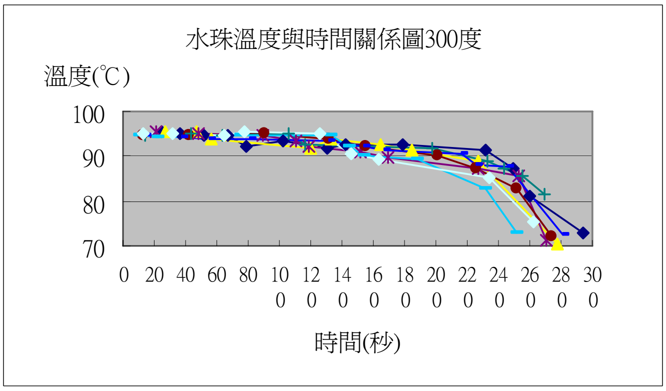
〈圖七〉不同滴數下的邊形數關係圖

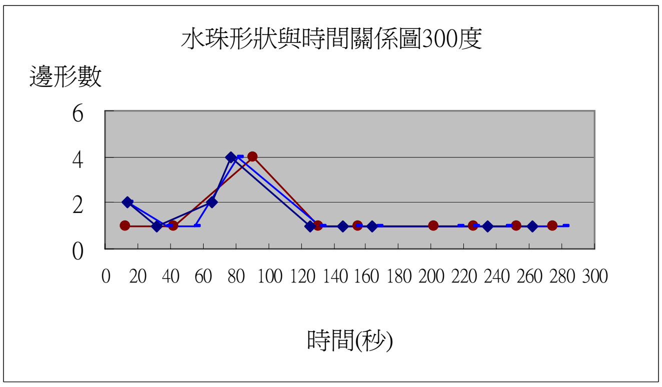
【實驗分析】:

於〈表七〉的 5 滴~20 滴數據中，都沒有出現橢圓形，但變形數仍會隨著滴數之增加而增加。而大於 20 滴的數據，則是常出現橢圓形旋轉，等到水珠變小時會以十字快速拉扯。我們更可以從〈圖七〉清楚的觀察到在板溫 350℃下是以 20 滴的變形數爲最多。

實驗五：水珠中心溫度及形狀變化與時間關係

步驟一：分別在 300℃、350℃、400℃的溫度下滴入 20 滴水珠(由實驗四之結論，我們決定以變形數最多的 20 滴來觀察)，並紀錄水珠變形過程中的中心溫度及變形過程，且分別用碼錶紀錄下水珠溫度當時的時間跟變形時之時間。

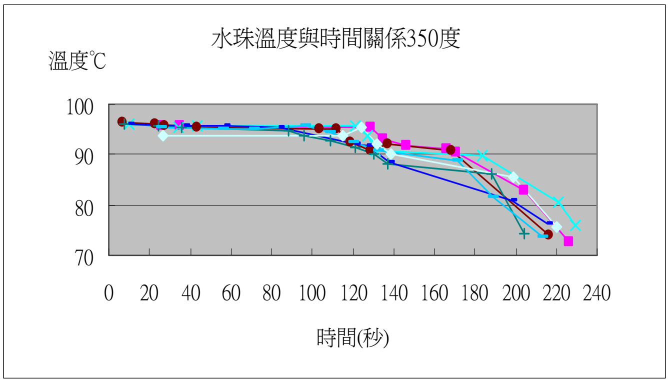


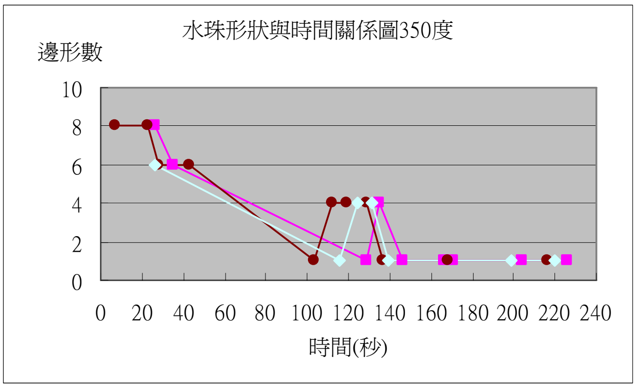


〈圖八〉水珠溫度與時間關係圖 300℃

＊ 水珠溫度與時間關係圖：水珠中心溫度先以平緩曲線下滑，到約２３０秒時開始驟降。

＊ 水珠形狀與時間關係圖：８０～１００秒時水珠變成十字形（以４表示）劇烈拉扯，之後則趨於穩定不再變形。

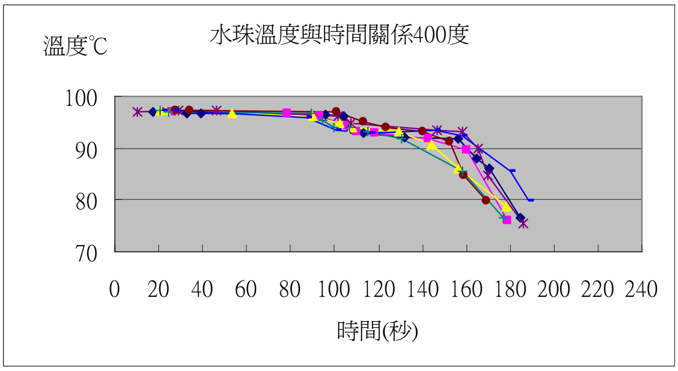


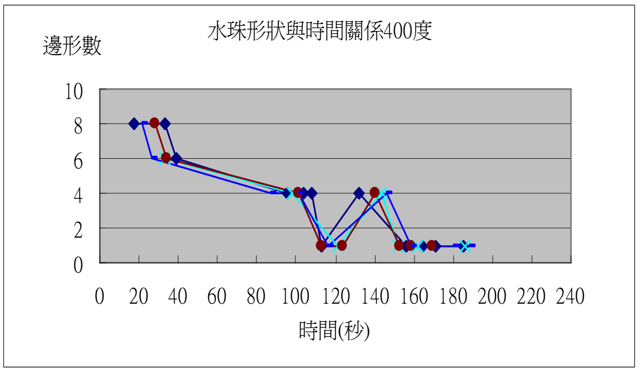


〈圖九〉水珠溫度與時間關係圖 350℃

＊ 水珠中心溫度與時間關係圖：水珠中心溫度於１２０～１４０秒之間明顯下降(此時十字形變回圓形)，並於１８０秒左右時驟降(此時為圓形)，直至水珠消失。

＊ 水珠變形規律：邊形數 8-6-1-4-1

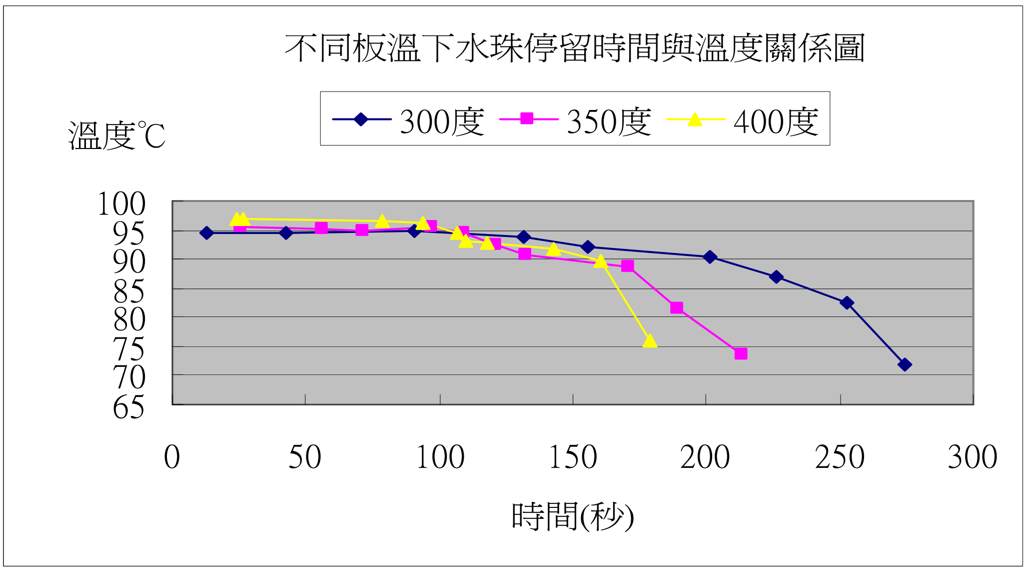




〈圖十〉水珠溫度與時間關係圖 400℃

＊ 水珠中心溫度與時間關係圖：水珠中心溫度於１００～１２０秒間明顯下降(此時十字變回圓形)，於１６０秒左右時驟降直至水珠消失(此時為圓形)。

＊ 水珠變形規律：邊形數 8-6-4-1-4-1



〈圖十一〉不同溫度下水珠停留時間關係圖

【實驗分析】:

於此實驗中可觀察到水珠中心溫度會隨著時間增加漸漸降低，此也說明了水珠中心溫度不會受到水珠變形的影響。除此之外，〈圖十一〉中水珠中心溫度一開始呈現穩定，到 100~200 秒之間會緩緩的降低，而後迅速降溫。也發現隨著板溫升高，水珠中心的初始溫度也較高，且停留時間較短，後階段的溫度驟降也較快。由對照的形狀與時間關係圖中可得知，隨著溫度的升高，變形也越趨複雜、多變，且變形規律皆由多邊形變至圓形。且當邊形數由多變少時，中心溫度會下降。

實驗六：水珠的平均半徑與邊形數關係

步驟一： 將高速攝影機之高度利用腳架固定且垂直拍攝高溫不銹鋼板

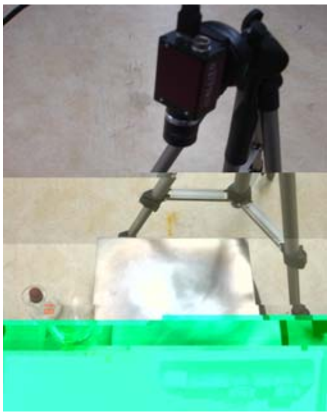
步驟二： 利用高速攝影機擷取水珠不同邊形數、不同大小之圖片

步驟三： 將高速攝影機之高度及垂直拍攝固定，現拍攝一張 1×1cm 之方格紙當作比例換算的工具。

步驟四： 將拍攝的圖片匯入 PhotoImpact 軟體，將圖片以 300% 的比例放大，放大後畫素不變，使用輪廓繪圖工具找出多邊形內切圓及外接圓之輪廓，固定 x 軸後利用測量工具找出 y 軸的畫素變化求出內切圓及外接圓半徑，將兩者平均求出平均圓之畫素。

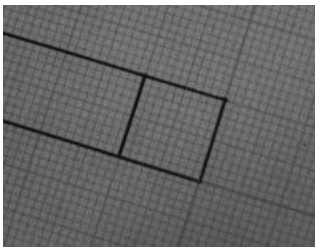
步驟五： 比較各種邊形數在不同加熱板溫下的平均圓半徑大小。

步驟六： 以方格紙所拍攝之 1cm：62.64 畫素 為比例尺換算出水珠的平均圓半徑約為多少 cm 並探討各種邊形數在不同板溫下的平均圓半徑的範圍。



〈圖十二〉

〈圖十三〉



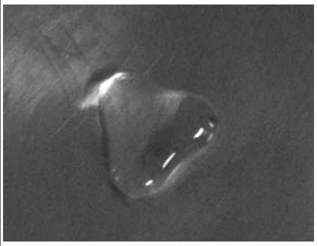
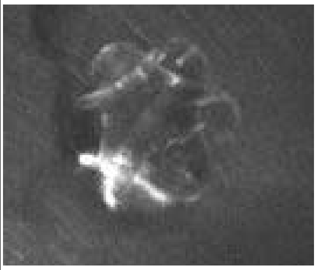
〈圖十四〉

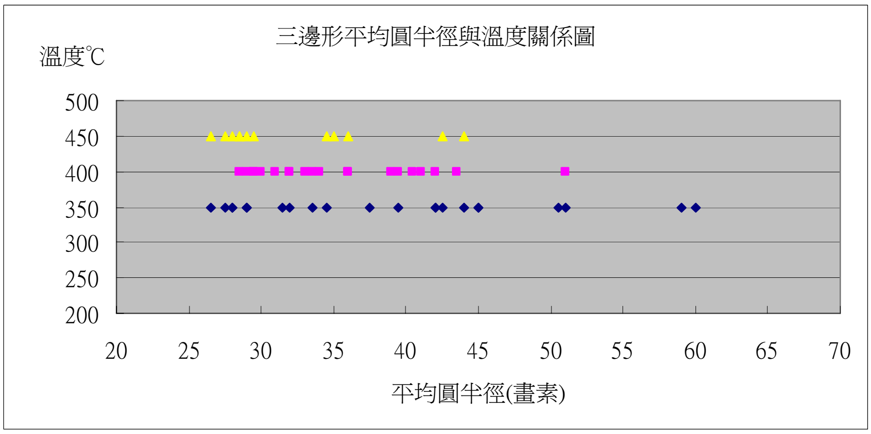
〈圖十二〉 觀察水滴變形時的實驗裝置(攝影機高度固定)

〈圖十三〉 攝影機高度固定下將不銹鋼板換成方格紙

〈圖十四〉 加粗的方格紙

〈表八〉三邊形　　　　　　　　　　　　　　　　　　　　　　　　（單位：畫素）

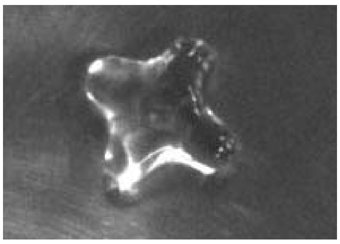
板溫 邊形數	450℃			400℃			350℃		
	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑
	36	52	44	40	62	51	40	62	51
	23	34	28.5	29	39	34	27	36	31.5
	36	49	42.5	29	38	33.5	40	50	45
	20	33	26.5	36	51	43.5	36	48	42
	29	43	36	28	44	36	20	38	29
	27	42	34.5	26	40	33	31	44	37.5
	22	31	26.5	23	41	32	40	61	50.5
	21	34	27.5	25	34	29.5	19	36	27.5
	22	37	29.5	23	36	29.5	32	47	39.5
	21	29	25	24	34	29	22	31	26.5

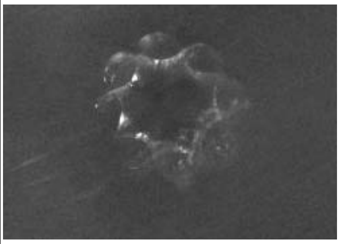


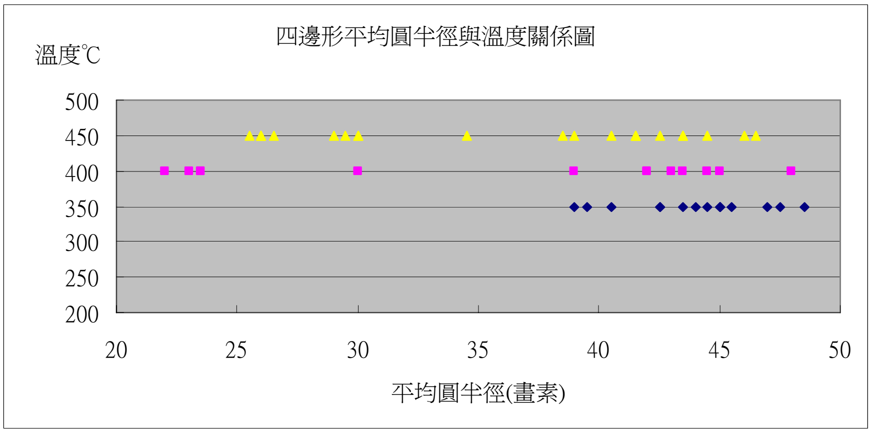
〈圖十五〉三邊形平均圓半徑與溫度關係圖

＊　三邊形的平均圓半徑在溫度較低時範圍較大，溫度愈高時範圍較小。且其半徑限定區之最大間隔隨著溫度之升高而變短。

〈表九〉四邊形　　　　　　　　　　　　　　　　　　　　　　　　（單位：畫素）

板溫 邊形數	450℃			400℃			350℃		
	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑
	36	59	47.5	36	60	48	42	55	48.5
	29	52	40.5	24	35	29.5	34	55	44.5
	33	60	46.5	26	34	30	30	51	40.5

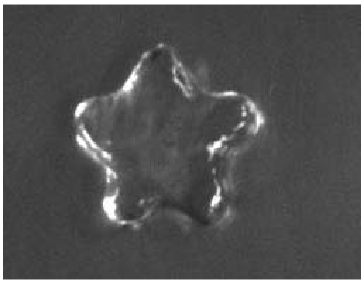
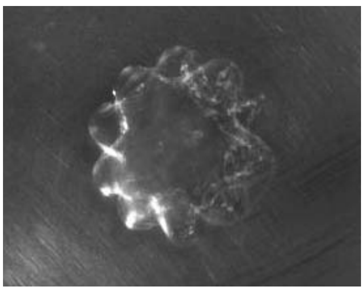
	31	56	43.5	35	55	45	31	54	42.5
	25	53	39	33	56	44.5	34	45	39.5
	28	49	38.5	25	33	29	41	54	47.5
	31	58	44,5	30	54	42	33	54	43.5
	29	54	41.5	28	50	39	34	51	42.5
	23	30	26.5	20	27	23.5	32	56	44
	22	29	25.5	19	25	22	35	43	39

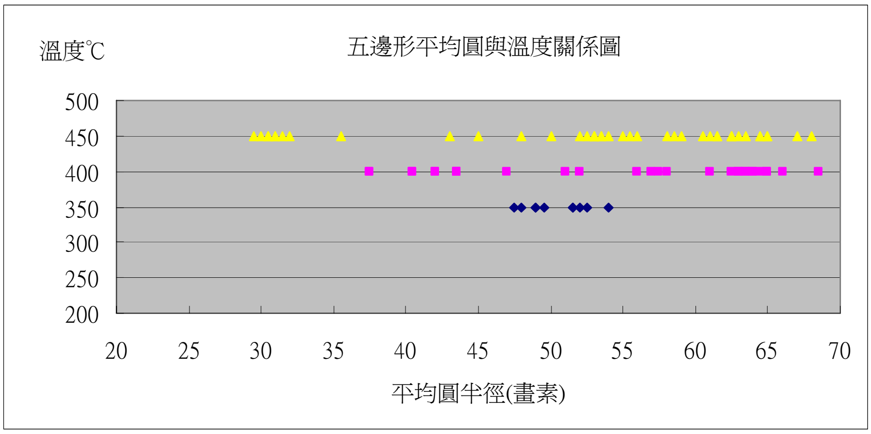


〈圖十六〉四邊形平均圓半徑與溫度關係圖

＊　四邊形之出現範圍以 400℃為最大，450℃次之，300℃最小。半徑限定區之最大間隔距離也依此順序減小。

〈表十〉五邊形　　　　　　　　　　　　　　　　　　　　　　　　（單位：畫素）

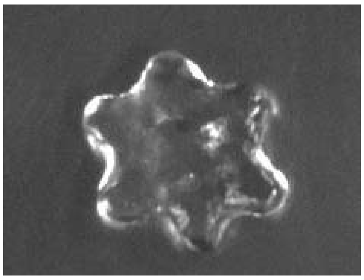
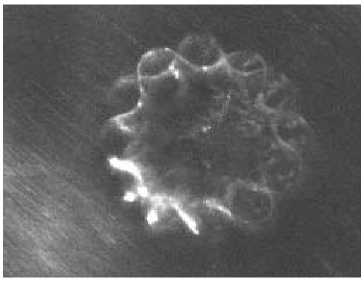
板溫 / 邊形數	450℃			400℃			350℃		
	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑
	59	77	68	54	83	68.5	61	84	72.5
	37	49	43	39	51	45	61	82	71.5
	50	76	63	38	46	42	44	61	52.5
	47	76	61.5	51	75	63	44	60	52
	45	73	59	47	68	57.5	46	62	54
	38	58	48	35	46	40.5	43	56	49.5
	39	51	45	54	75	64.5	43	53	48
	45	62	53,5	49	74	61.5	41	57	49
	20	25	22.5	40	59	49.5	44	59	51.5
	21	24	22.5	34	41	37.5	41	54	47.5

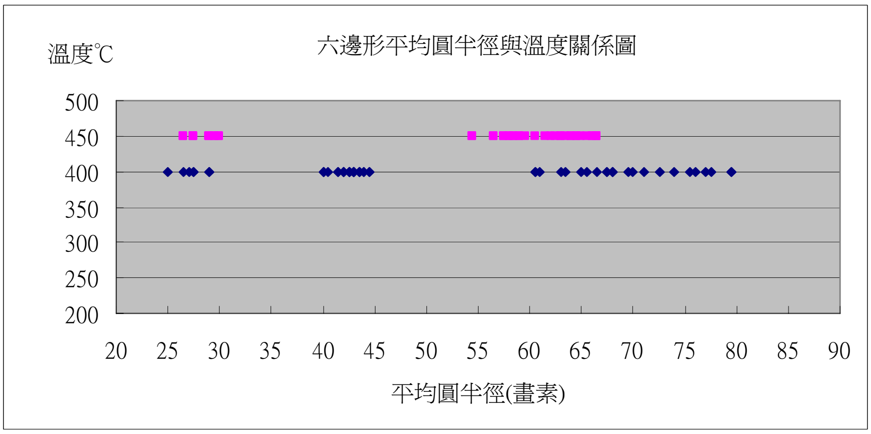


〈圖十七〉五邊形平均圓半徑與溫度關係圖

＊ 溫度越高，平均圓半徑範圍越大，水珠半徑限定區的最大間隔也越大。

〈表十一〉六邊形　（單位：畫素）

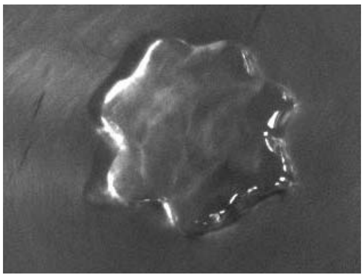
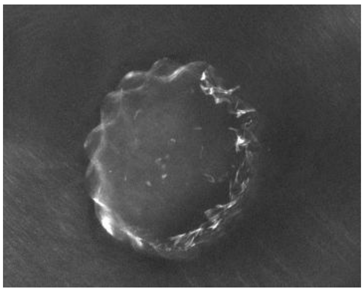
板溫 邊形數	450℃			400℃			350℃		
	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑
 	54	79	66.5	70	89	79.5			
	54	75	64.5	56	80	73			
	52	74	63	54	81	67.5			
	53	70	61.5	49	72	64.5			
	48	69	58.5	37	52	44.5			
	50	59	54.5	37	49	43			
	25	35	30	35	45	40			
	24	34	29	36	49	42.5			
	24	31	27.5	23	32	27.5			
	23	30	26.5	22	28	25			

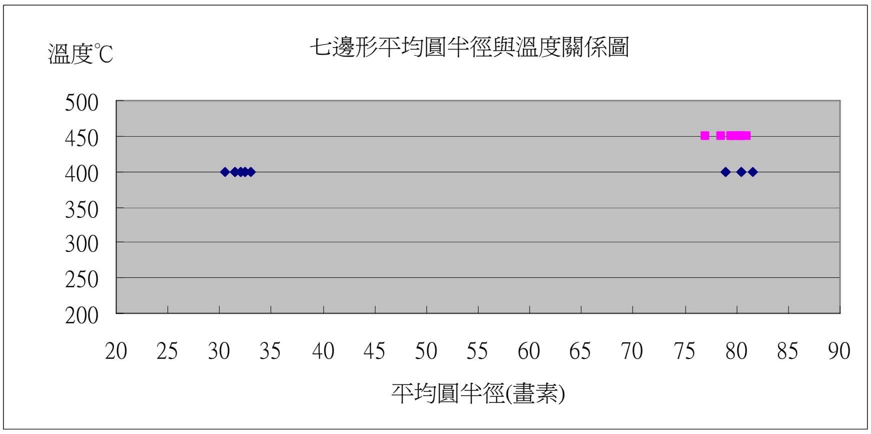


〈圖十八〉六邊形平均圓半徑與溫度關係圖

＊ 溫度越低，平均圓半徑範圍越大。但４５０℃時水珠半徑限定區的間隔較大。

〈表十二〉七邊形　（單位：畫素）

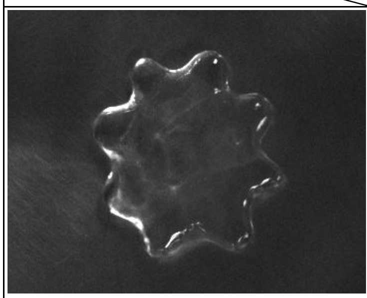
板溫 / 邊形數	450℃			400℃			350℃		
	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑
	72	91	81.5	71	92	81.5			
	65	92	78.5	72	89	80.5			
	69	92	80.5	71	87	79			
	64	90	77	70	91	80.5			
	69	93	81	71	87	79			
	70	90	80	30	35	32.5			
	71	90	79.5	31	35	33			
	69	89	79	30	34	32			
	68	88	78	30	33	31.5			
	64	89	76.5	29	32	30.5			

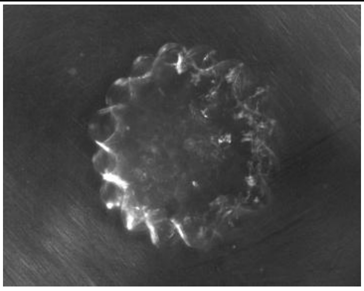


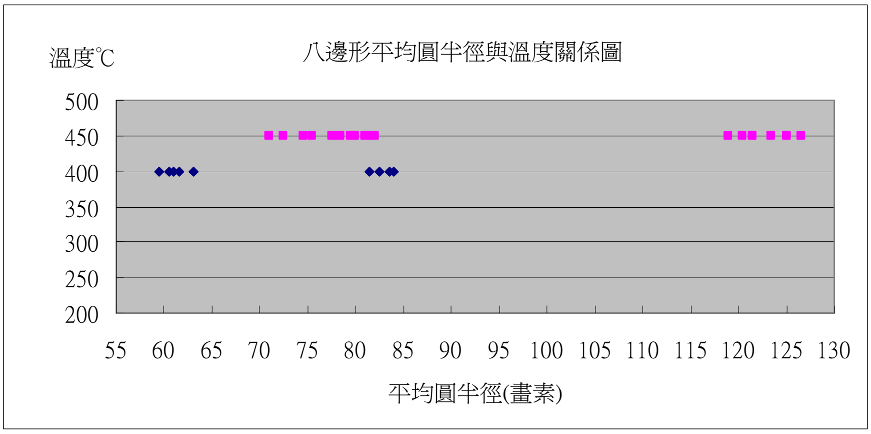
〈圖十九〉七邊形平均圓半徑與溫度關係圖

＊ 此圖４００℃時的半徑限定區間隔很大，因此我們推測４５０℃的水珠在更大半徑時會出現其它的半徑限定區。

〈表十三〉八邊形　（單位：畫素）

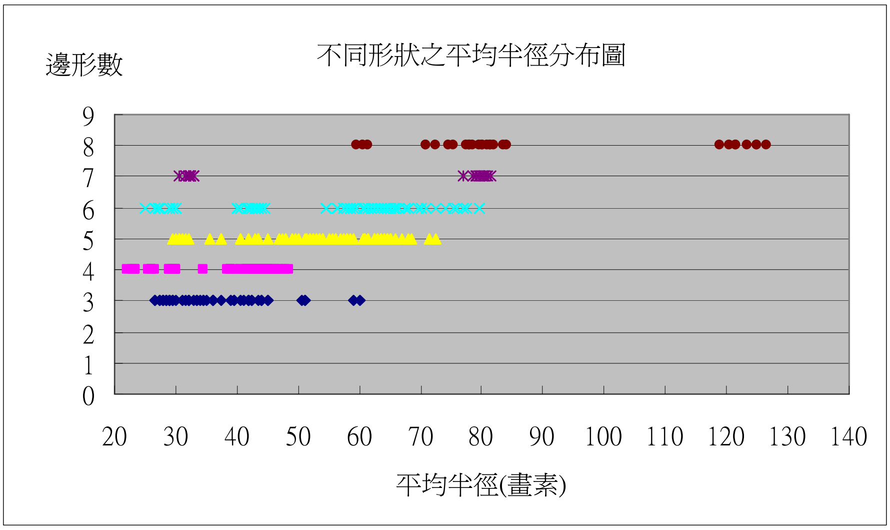
板溫 / 邊形數	450℃			400℃			350℃		
	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑	內切圓半徑	外接圓半徑	平均半徑
	63	190	126.5	75	93	84			
	72	91	81.5	56	67	61.5			
	66	184	125	57	64	60.5			
	68	88	78	54	65	59.5			

	71	88	79.5	74	93	83.5	
	71	89	80	71	92	81.5	
	64	183	123.5	72	93	82.5	
	67	171	119	56	66	61	
	73	91	82	58	68	63	
	61	81	71	54	65	59.5	

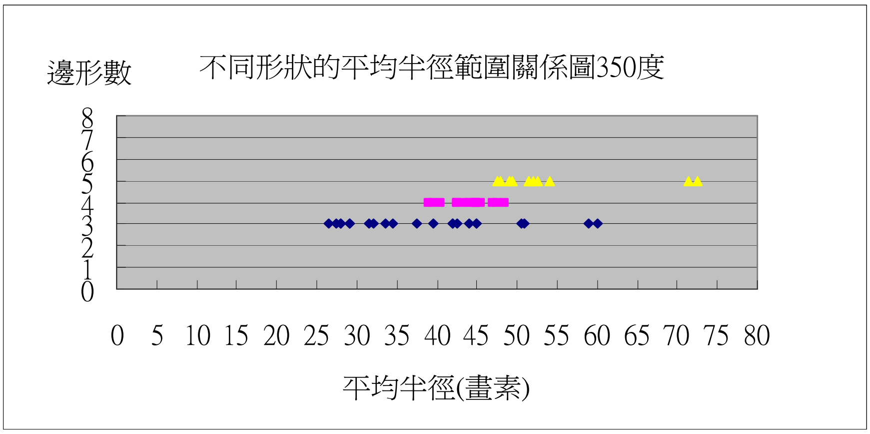


〈圖二十〉八邊形平均圓半徑與溫度關係圖

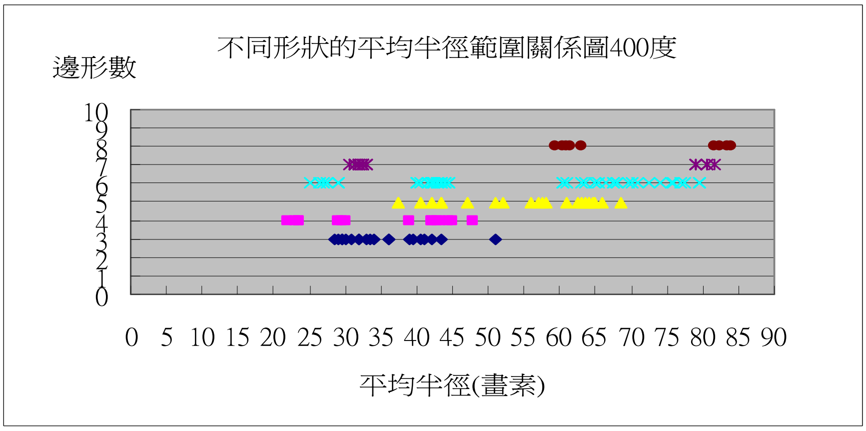
＊ 溫度越高，半徑範圍越大，且其半徑限定區的間隔也越大。



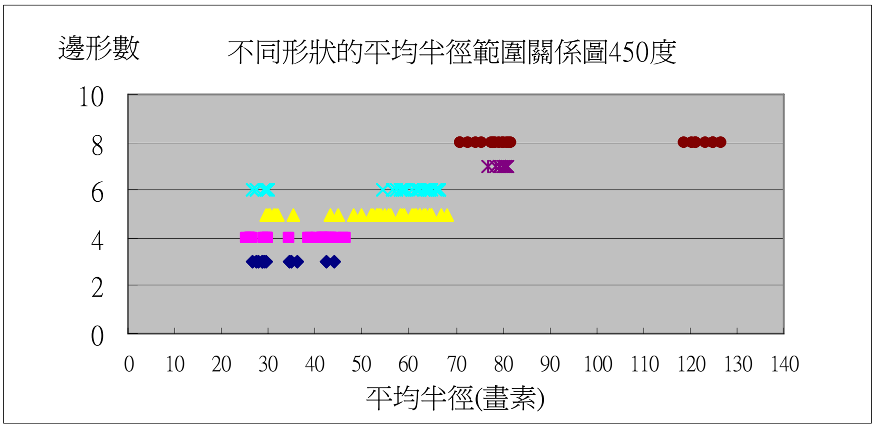
〈圖二十一〉不同形狀之平均半徑分布圖



〈圖二十二〉不同形狀的平均半徑範圍關係圖 350 度

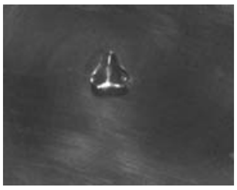
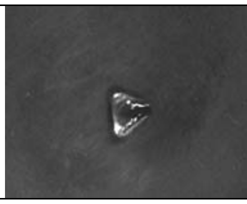
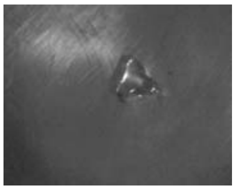
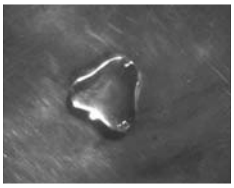


〈圖二十三〉不同形狀的平均半徑範圍關係圖 400 度

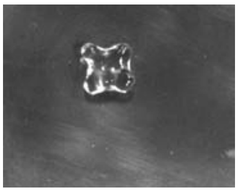
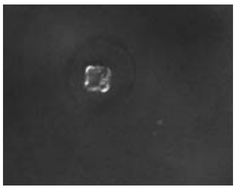
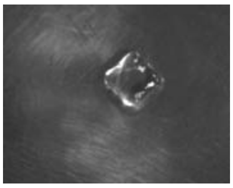
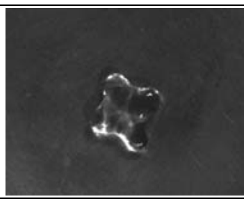


〈圖二十四〉不同形狀的平均半徑範圍關係圖 450 度

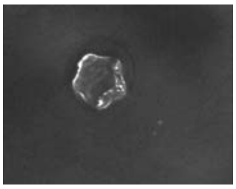
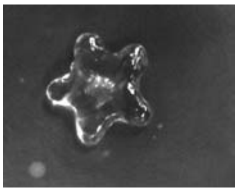
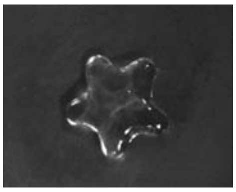
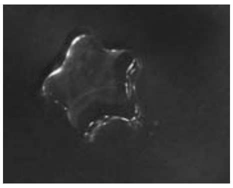
〈表十四〉各種形狀在不同板溫下出現之最大及最小平均半徑比較 (單位：公分)

板溫	450℃		400℃		350℃	
最小平均半徑(cm)		0.4		0.46		0.42
最大平均半徑(cm)		0.7		0.81		0.81

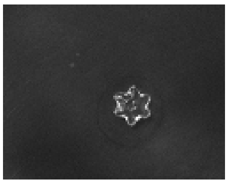
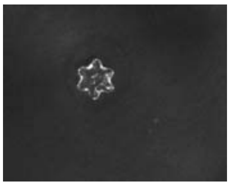
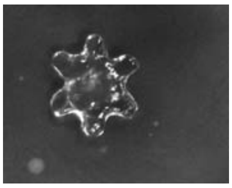
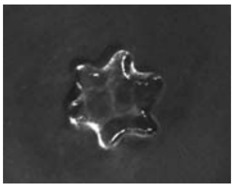
〈表十五〉

板溫	450℃		400℃		350℃	
最小平均半徑(cm)		0.41		0.35		0.62
最大平均半徑(cm)		0.76		0.77		0.77

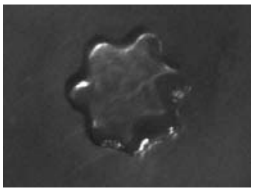
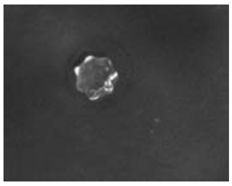
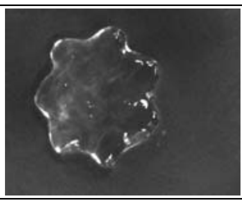
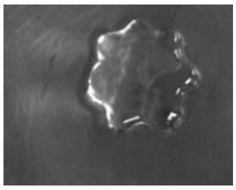
〈表十六〉

板溫	450℃		400℃		350℃	
最小平均半徑(cm)		0.36		0.6		0.76
最大平均半徑(cm)		1.09		1.09		1.16

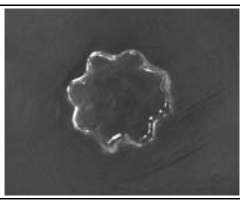
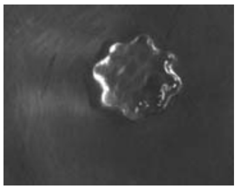
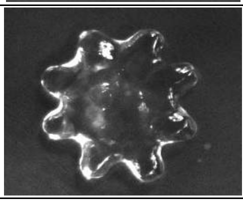
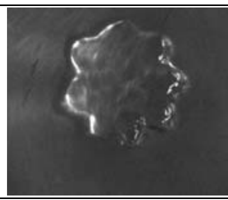
〈表十七〉

板溫	450℃		400℃	
最小平均半徑(cm)		0.42		0.4
最大平均半徑(cm)		1.06		1.22

〈表十八〉

板溫	450℃		400℃	
最小平均半徑(cm)		1.23		0.49
最大平均半徑(cm)		1.30		1.25

〈表十九〉

板溫	450℃		400℃	
最小平均半徑(cm)		1.13		0.95
最大平均半徑(cm)		2.02		1.34

【實驗分析】:

1. 由（圖二十一）中發現，除了四邊形以外，當邊形數較高時，平均圓半徑也較大，但不同邊形數之間的平均圓半徑範圍有部分的重疊。
2. 由（圖二十二、二十三、二十四）中發現，隨著板溫之升高，多邊形的平均半徑範圍增大，但低邊形的範圍則變小。反之，板溫較低時，低邊形的半徑範圍大，多邊形則小。

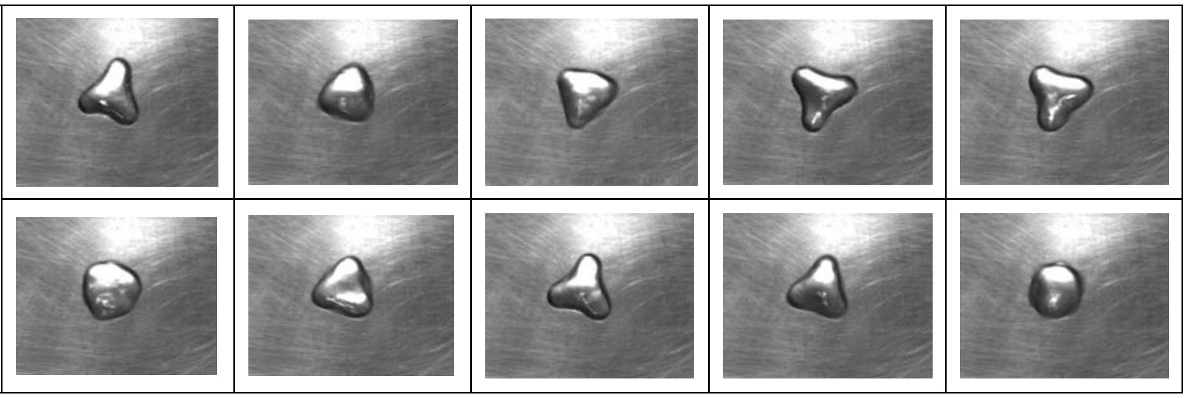
柒、實驗討論：

一、水珠的基本圖形及變化規律

實驗一〈表四〉中，水珠在突出變形與表面積縮小之下，劇烈變形後恢復球形。如此交替循環變化，如簡諧振盪一般。

因此我們推測：水珠處高溫狀態急待蒸散熱能，所以水珠增大表面積加快蒸發速率，但水珠變形的同時與 Hotplate 接觸表面積變大，單位時間熱傳導能量增加，因此又變回球形，如此交替變化。

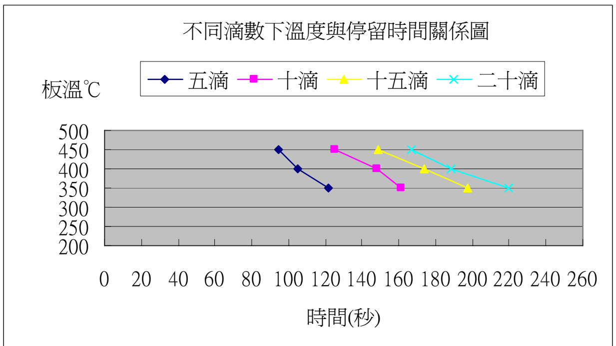
三角形變形規律



二、水珠停滯時間在不同加熱板溫度下之探討

實驗二〈圖三〉中，水珠的停滯時間會隨著板溫的降低而延長。且水珠除了隨著水滴數之增加而停滯時間延長外，也會隨著水滴數之增加，在相同板溫下兩兩滴數(如５滴和１０滴之間)之間的停滯時間差值越小。

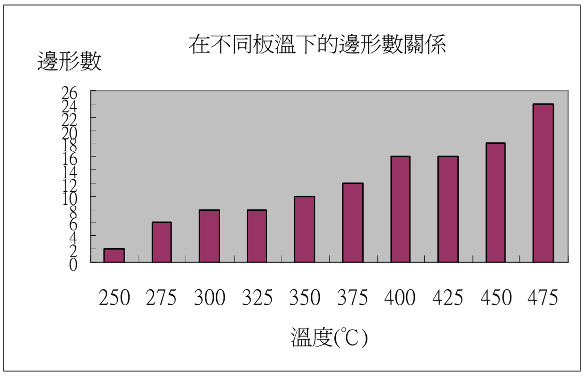
因此我們推測：隨著板溫之升高，水珠蒸發的速率也越快，且板溫對水珠停滯時間之影響大於水珠本身的大小(滴數)。



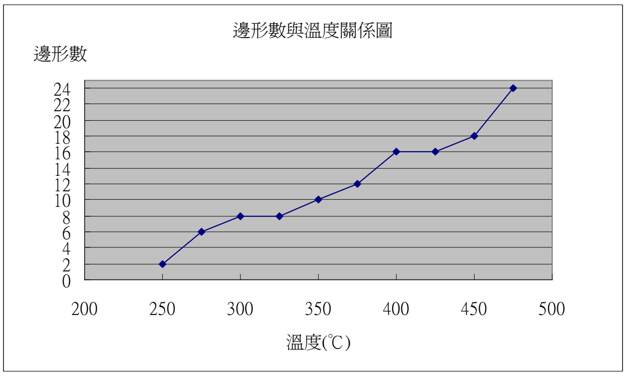
三、水珠於不同加熱板溫度下之變形數關係

實驗三〈圖五〉中我們發現：隨著溫度的升高，邊形數也隨之增大。此外， 8 邊形跟 16 邊形在一段溫度下都不會再增加邊形數。

因此我們推測：水珠隨著板溫之升高而變出更多邊形以增加表面積散熱，同時也以振盪減小吸熱面積，達到最佳的散熱機制。其中某些邊形的出現需要更多的熱能，所以會形成一段熱能差，持續增多邊形數。如（圖六）中８→１０、１６→１８即是如此。



(圖五)



(圖六)

四、不同滴數與變形數之關係

實驗四的〈表七〉5 滴~20 滴數據中，都沒有出現橢圓形，且變形數會隨著滴數之增加而增加。而大於 20 滴的數據，則會出現橢圓形旋轉，等水珠變小時改以十字快速拉扯。

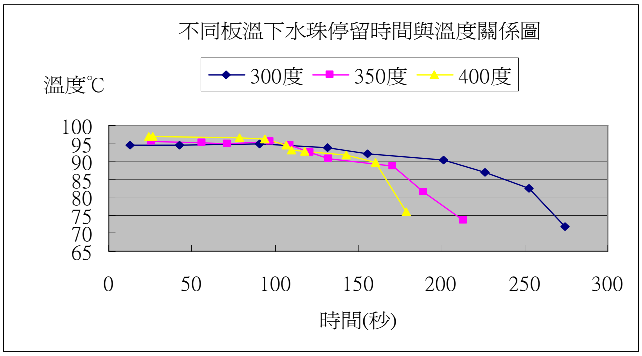
因此我們推斷：在滴數低於 20 滴時，由於水珠較小，所以會以快速的十字形振盪取代橢圓形旋轉。當水珠大於 20 滴時，由於水珠太大，而板溫又不夠高，所以沒有足夠的熱能形成十字型，故一開始只會以橢圓形旋轉以散熱，等水珠變小時，則會改以十字形振盪散熱。

滴數＼邊形數	圓形 (1)	橢圓形 (2)	十字形 (4)	三邊形 (6)	四邊形 (8)
5 滴	*		*		
10 滴	*		*		
15 滴	*		*	*	
20 滴	*		*	*	*
25 滴	*	*	*		
30 滴	*	*	*		

五、不同溫度下水珠停留時間關係

實驗五我們觀察到水珠中心溫度會隨著時間緩緩降低（由９７℃～７１℃），而且變形數並不會影響水珠之中心溫度。水珠中心溫度到 100~200 秒時會緩緩的降低，而後迅速降溫。隨著板溫升高，水珠中心的初始溫度也較高，且停留時間較短，溫度下降較快。同時也發現水珠變形由多邊形數降至低邊形數的規律變化（８→６→４→１→４→１）

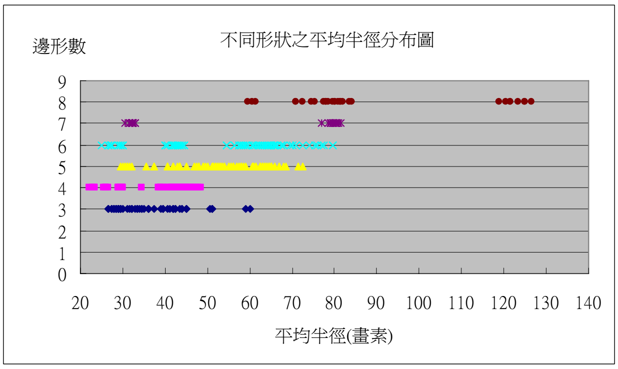
因此我們推測：水珠變形的目的在於降低溫度、散發熱能。所以板溫越高、擁有之熱能越多，變形數也隨之增加以達劇烈拉扯、轉動，好將過多的熱能散發出來以降低水珠中心溫度達到穩定、平衡。所以水珠的中心溫度並不會受到變形而影響。但板溫之高低確實會影響水珠初始跟最終的溫度。



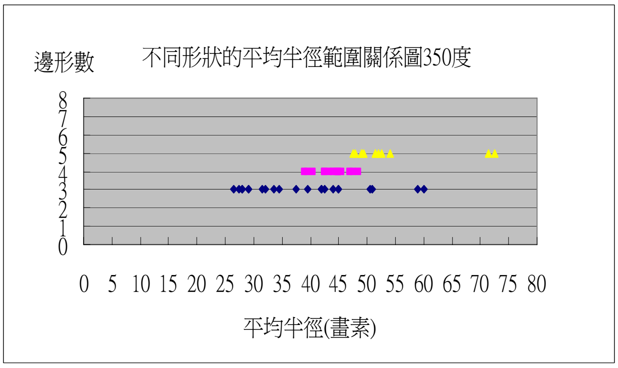
六、各種形狀的平均圓半徑與溫度關係

1.由實驗六（圖二十一）中發現，除了四邊形重影外，當邊形數較高時，平均圓半徑也較大，但不同邊形數之間的平均圓半徑範圍有部分的重疊。

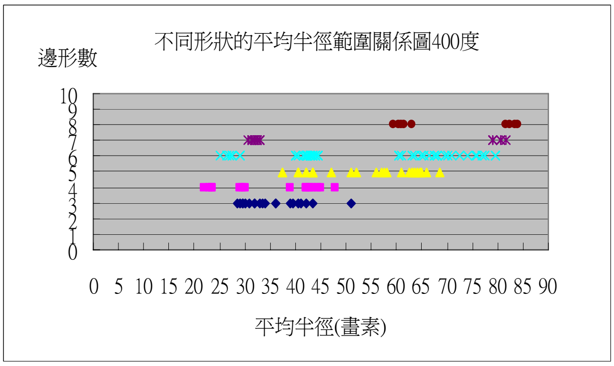
2.由（圖二十二、二十三、二十四）中發現，隨著板溫之升高，多邊形的平均半徑範圍增大，但低邊形的範圍則變小。反之，板溫較低時，低邊形的半徑範圍大，多邊形則小。



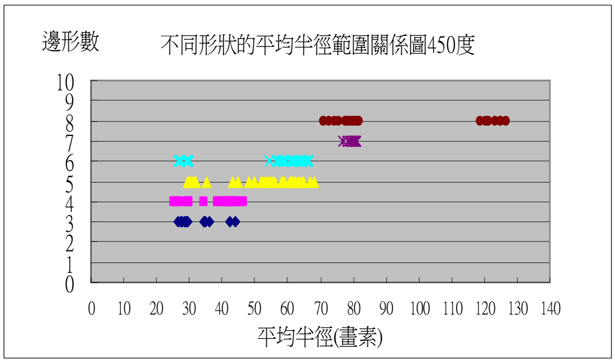
(圖二十一)



(圖二十二)



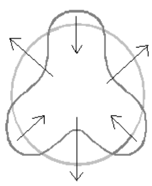
(圖二十三)

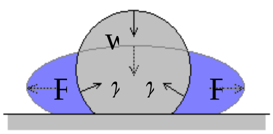


(圖二十四)

捌、結論

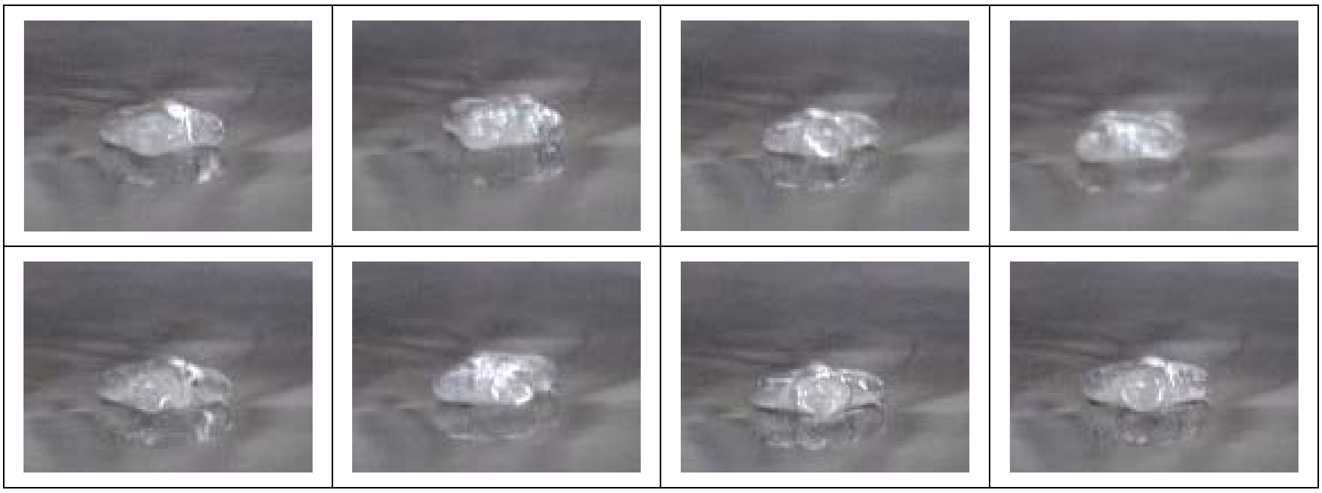
1. 一般液體之黏度、表面張力皆因溫度上升而降低。由於表面張力內縮作用，表面積變化要最小，水珠在突出變形與表面積縮小之下，劇烈變形後恢復球形，突出一角的相鄰爲內凹並以參考圓爲準，如此交替循環變化，如簡諧振盪一般，將熱能以位能和動能形式互換，對稱變形以達最小能量吸收，且以轉動方式在擾動之下，取得動態穩定平衡達到最快的蒸發。



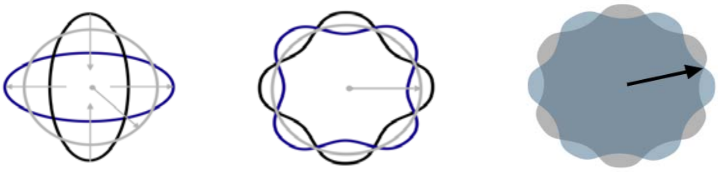


2. 由實驗原理觀念一得知：常溫水珠的水分子因為電偶極及高溫熱分子運動，水分子在蒸氣膜上產生劇烈的運動以增加表面積來散熱；同時也藉由降低表面積減少熱由 Hotplate 傳導至水珠，以達最小吸收，最快蒸散的目的，水分子形狀因此就依 Laplace’s equation 所得之結果不停的震盪與變形。$r(\theta,\phi) = R + \sum c_n \cos(\omega_n t) P_n(\cos\theta)$

〈表二十〉 四邊形重影側面圖



3. 水珠的基本形狀有圓形、橢圓、三邊形、四邊形、五邊形、六邊形、七邊形、八邊形、九邊形、十邊形、十一邊形、十二邊形。同時加熱板上，溫度越高，形狀變化種類較多（變形數），最大變形數高，且自轉速度較快，停滯時間變短。水滴越大的形成水珠時，變形越豐富種類較多，自轉速率因重力而降低，但停滯時間越久。總之水滴大小會影響最大變形數，也就是最大變形數由水滴大小及溫度高低決定，即邊形數隨溫度之升高及滴數之增加而變大。
4. 水珠在特定邊形數〈如：8 邊形(300℃~350℃)與 16 邊形(400℃~450℃)〉須等到吸收或散出更多的熱量，才能完成下一個形狀的轉變，因此水珠在一段較大的溫度範圍內均不再增加邊形數。此過程與水的『潛熱』有略同之處。
5. 由水珠平均圓半徑的實驗作深入的水珠大小分析統計，發現有數據顯現水珠某一邊形數與不同加熱板溫下，水珠平均圓的半徑範圍有一定限制。隨著板溫之升高，多邊形的平均半徑範圍增大，但低邊形的範圍則變小。反之，板溫較低時，低邊形的半徑範圍大，多邊形則小。平均圓半徑並非連續性的，此現象表示特定的能量能形成特定的邊形數，且有特定的平均圓半徑限制，此現象與化學上所討論的電子組態結構的模型極為類似，亦即水珠變形的能量也遵守能量量子化。



玖、感想與展望

感想：

在水珠溫度的測定中，依我們的觀測及思考判斷，認為水溫低時邊形數少，水溫高時邊形數多。可是經過數位溫度計的測量後，深深體會到實驗假設及思考判斷和實驗測量結果有時差距是很大的，因此我們確知科學實驗中必須有精密的測量，才是真正的科學研究。

重測再重測、修正再修正、驗證再驗證，花費了我們許多的心血，但當報告成形時，這一切的辛苦都是值得的，在科展研究過程中，我們學會了多方面的思考、追求完美的執著、虛心求教的態度，水珠完美的形狀變化就如我們精采豐富的結果。

展望：

從空中灑下黃金似的水滴，於炙熱板上卻是冰圓玉潤。於蒸氣膜上因表面張力及電偶極性作用的效應，水珠多種面貌、多樣狀態不停的變化演出世界生命的永恆。本作品是緣起水珠漫舞之源，卻是水珠律動機制之泉。小小一滴水，有永遠做不完的研究。

水珠多樣對稱變形，訴說著能量變形轉動互動機制。水珠的能態總是在圓周長為律動波長整數倍的駐波形式下，達動態穩定平衡。時而震盪變形，時而轉動變形。是自然界中美妙的圓駐波能量量子化實例。

拾、參考文獻

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二、　作者：黃定加　物理化學　－高立圖書有限公司（第四章：熱力學第二、三定律　第十章：界面化學）

三、　作者：林明瑞　高二物質科學物理篇下冊：第九章：流體力學、第十章：熱

四、　作者：楊宗翰　清大工科系碩士論文－微液膜蒸發的分子動力學模擬

五、　By Morihiro Okada・Minoru Okada　Exp Fluids(2006) 4:789-802 Observation of the shape of a water drop on an oscillating Teflon plate.

評語

變因的控制相當準確，實驗現象的擷取也非常清晰。故，結論對自然現象的描述完整而易於推論，殊勘嘉獎。

A. Introduction

A1. Motivation

When a Japanese teppanyaki chef pours some water on the hotplate, it is interesting to see that some of the water does not evaporate immediately. Some of the water forms droplets and bouncing on the plate between the delicious foods. Motivated by this observation, we decided to use a simple setup to reproduce and examine these interesting phenomena, and tried to understand the science of these dancing water droplets.

A2. Other's work

Holter used a hotplate and observed the behavior of the droplets on a heated plate in 1954 [1]. Later, in Morihiro Okada and Minoru Okada's research [2]" Observation of the shape of a water drop on an oscillating Teflon plate", they used a oscillating Teflon plate to vibrate the droplets in the room temperature. The behavior of the droplets on the Teflon plate was similar to our observations described below. The droplets would also oscillate and transform into various shapes. However, droplets in our experiments oscillate with their own natural frequencies and in various temperatures. In Morihiro Okada and Minoru Okada's study, the oscillation frequency of the droplets was based on the principle of Rayleigh's equation [2]. Rayleigh's equation is used to describe the relation between the oscillation frequencies and the radius of a sphere. However, we discuss water droplets with various shapes. Gravity plays an important role in our cases.

A3. Background

The Leidenfrost effect is a phenomenon in which a liquid, in close contact with a mass significantly hotter than its boiling point, produces an insulating vapor layer which keeps the liquid from boiling rapidly. It was named after Johann Gottlob Leidenfrost, who discussed it in "*A Tract About Some Qualities of Common Water"* in 1756 [3]. We can easily observe the Leidenfrost effect in our experiment. As the temperature of the hotplate goes above 100 °C, the water drops hiss on touching the pan and evaporate relatively quickly. Later, as the temperature goes past the Leidenfrost point, the Leidenfrost effect comes into play. This time, they bunch up into small balls of water and lasting much longer. This is because, at temperatures above the Leidenfrost point, when water touches the hot plate, the bottom part of the water vaporizes immediately on contact. The resulting vapour actually suspends the rest of the water droplet just above it, preventing any further direct contact between the liquid water and the hot plate and dramatically slowing down further heat transfer between them. This also results in the drop being able to skid around the the hotplate on the layer of gas just under it. The temperature at which the Leidenfrost effect begins to occur is not easy to predict. Even if the volume of the drop of liquid stays the same, the Leidenfrost point may be quite different with a complicated dependence on the properties of the surface as well as any impurities in the liquid. The Leidenfrost point may also be taken to be the temperature for which the hovering droplet lasts longest.

A4. Our work

The motions of water droplets on top of a hotplate have been studied using a high speed camera. Our experiment is not only the observation of different shapes of the moving water droplets, but also the analysis of their frequencies in different shapes and radius. We used a simple setup to find out the conditions such as temperature of the hotplate and the volume of the droplet for the formation of an oscillating drop. In addition, the relation between the shapes of droplets and their oscillation frequencies, radius and the temperature of the plate are studied. As will be shown below, our data can be described by a form derived by Rayleigh for an oscillating sphere. We also find that there are two oscillating modes for a droplet with the same radius and same shape. The latter observation might be related to period doubling route to chaos [4]. Finally, the Leidenfrost effect has been verified in our experiment and can be described by lubrication theory [5].

A5. Goals

The scientific goals of our studies are:

1. To find out how temperature and size affect the formation of a droplet and to investigate the lifetime of a droplet under various conditions.
2. To investigate the shapes of the droplets and their relationship to the oscillation frequencies, radii and temperatures.

B. Experiment

The setup of the experiment is very simple, as showing in Figure 1. It is consisted of a power controlled hotplate and a stainless steel plate. Temperature of the hotplate is measured both by thermal couples and an infra-red camera.

B1. Apparatus

The following is the list of the equipments:

1. Stirrer Hotplate(Corning)
2. Fujinon corporation 1:1.4/16mm HF16HA-1B
3. Dropper and Erlenmeyer flask
4. The venier caliper
5. A homemade kickstand
6. Stainless plate×2 (19.7cm×16.3cm×0.1cm)
7. Thermal couples (K/J type)
8. Stopwatch

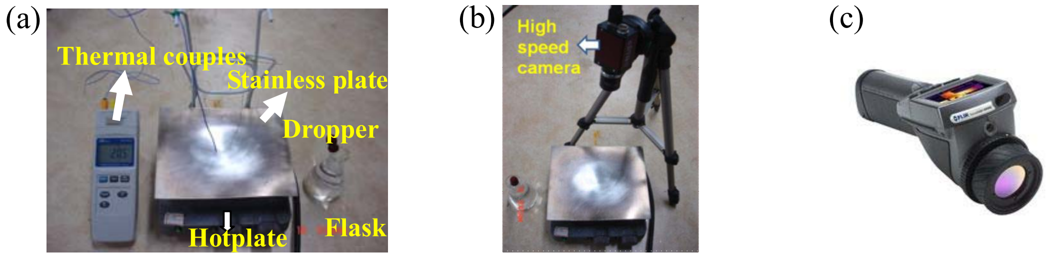


Figure 1. (a) The basic setup in the experiment. (b) A high speed camera. (c) An Infrared camera

To start an experiment, the hotplate is first setup to a target temperature by the power control and the measurement from thermal couples. Next, a known amount of water is added to the hotplate by a dropper. The motion and the shapes of the droplets on the hotplate are then recorded by a high speed camera. The following is the experimental procedure:

B2. Experimental Procedure:

1. Modification of the stirrer hotplate: The original ceramic plate of the hotplate was removed and then a stainless plate (19.7cm×16.3cm×0.1cm) was attached on the heater to create the condition for teppanyaki plate, as shown in Fig.1 above.
2. A thermal couple (K/J type) is used to measure the temperature of the plate which can be controlled.
3. The volume of the droplet is controlled by the number of drops of the distilled water delivered by a calibrated dropper.
4. The lifetime of the water droplets on the hotplate is recorded using the stopwatch.
5. Using a high speed camera, we can record and analyze the motion of the droplet frame by frame, the way water droplets oscillate and also the oscillation frequencies of the droplets.

6. By calibrating the pixel size of the frame with a venier caliper, we can measure the size of the droplets using the computer software Photo Impact.
7. The temperature of the droplet is monitored by an infra-red camera and thermal couples.

B3. Digital Image Processing:

The images captured by the high speed camera contain the information for the vibration and shapes of the droplets. The software "Photo Impact" is used to measure the physical parameters of the droplets such as: its radius and vibration frequencies. The following is the procedure for such measurements:

a) Measurement of droplets radii

1) Use a high speed camera and take pictures of the oscillating droplets on top of the hotplate at a fixed position.
2) Then take another picture of the hotplate with a ruler on top of it at the same position as in Step1).
3) By the help of the software "Photo Impact", two circles can be drawn on for a droplet as shown (R1 and R2). where R1 and R2 corresponds to the larger and the smaller circles as shown in Fig. 2. We define the radius of the droplet as R'= (R1+R2)/2. The definition of the water droplet is depicted in Fig. 2.
4) R can be converted from pixels to cm by the picture of the ruler taken in Step 2).

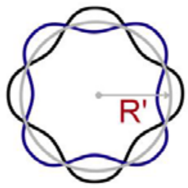


Figure 2. The definition of radius.

b) Measurement of the droplets' oscillation frequency

1) Control the frame rate of the high speed camera to a proper speed.
2) Use the high speed camera to record a series of pictures of the oscillation drop. A typical oscillation is illustrated in Fig. 3.
3) Count the number of the photos of a dancing drop for one period.
4) The period or the frequency of the oscillation can be calculated by using the known frame rate.

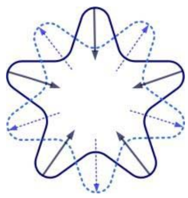


Figure 3. A typical oscillation of a water droplet.

B4. Summary of Definitions used in this experiments

Temperature(T): The temperature of the heated plate.
Lifetime: The time the droplet stays on the heated plate
R: Radius of a spherical droplet. (Fig. 4, left).
R': Radius of flat droplets. (Fig. 4, left).
n: The number of the lobes of different shapes.
Frequency: The oscillation frequency of the droplets (sec^{-1}).
e: The thickness of the vapor layer. (mm)

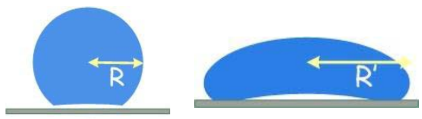


Figure 4. The definitions of radii for the two typical shapes of the drop..

The working principle of our experiment can be summarized schematically by the following diagram (Fig. 5):

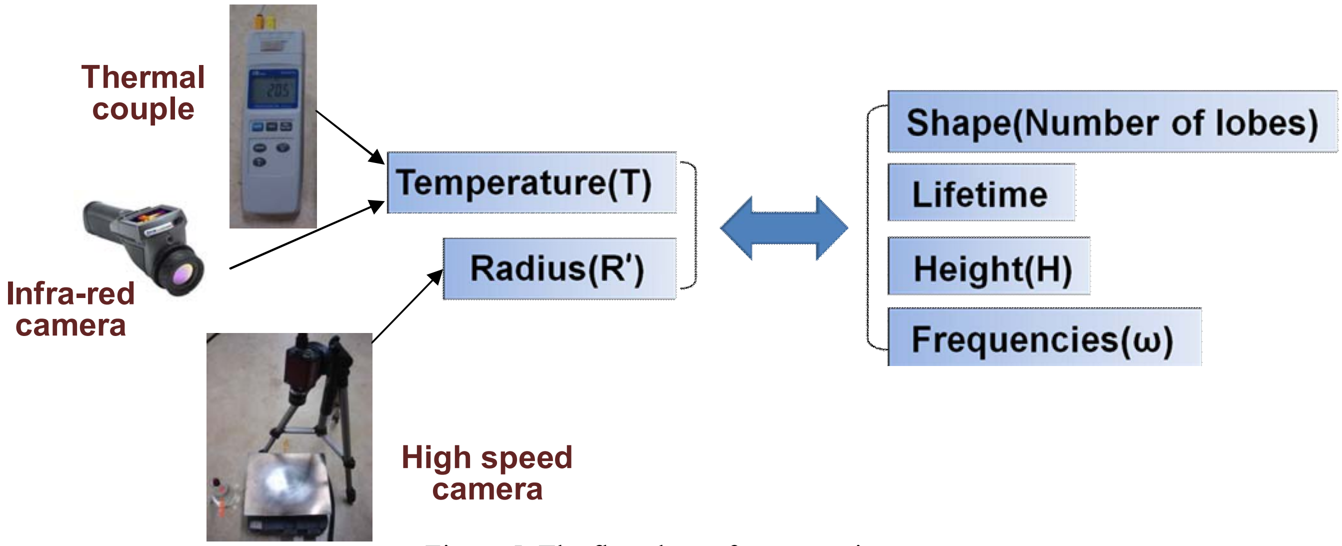
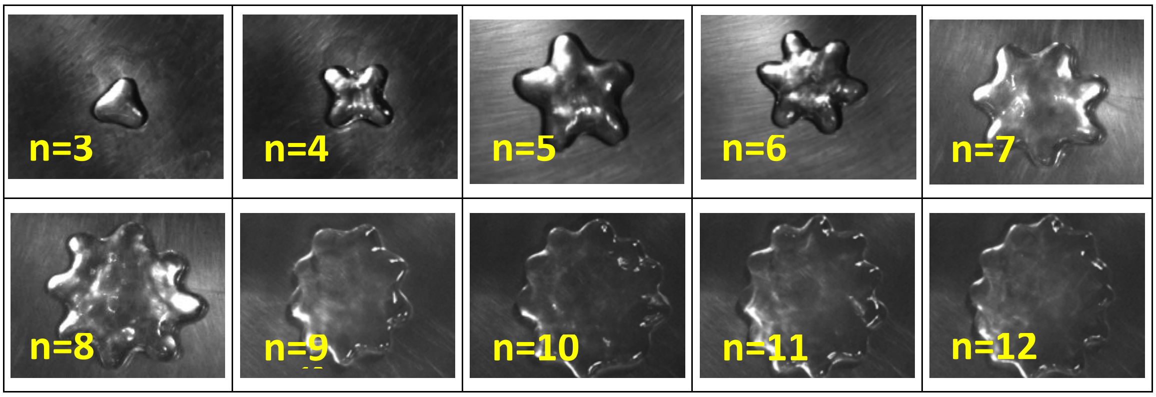


Figure 5. The flowchart of our experiment.

C. Results:

C1. Observations of the different Shapes



n = number of lobes
Figure 6. Different shapes observed with different size.

Using the high speed camera, we first found that the droplets on the hotplate will not just bounce around. In fact, if a bigger droplet were added onto the hot surface, the droplets would transform into non-trivial, but regular shapes. As can be seen in Fig. 6, the larger the size of the droplet, the more lobes it contains. The size of the lobes seems to be more or less constant, quite independent of the size of the drop. This observation leads to the measured linear relationship between the drop radius and the number of lobes. The photos below were taken by high speed camera (Fig. 6)

C2. Details of the oscillatory motions

Detailed motion of the droplets can be studied using a high speed camera. In the followings, we take pentagon as an example. We observed that the droplet not only change their shapes but also oscillate in time. When the lobes stretch out, the center of the droplet will descend and vice versa. In addition, there are fluctuations on the surface of the droplet while the droplet draws back its lobes to form a more spherical shape. (Fig. 7)

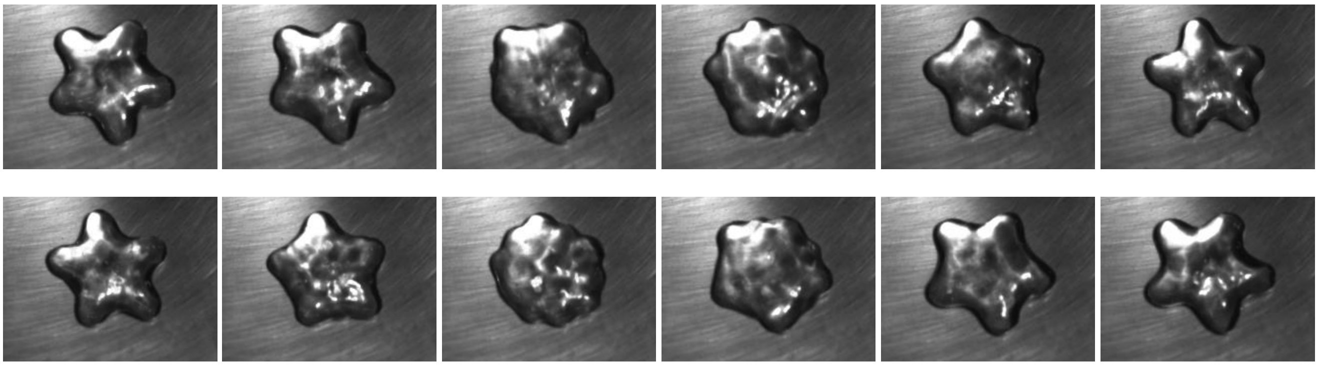
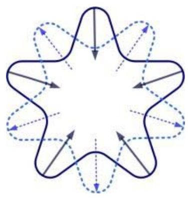


Figure 7. A typical droplet showing a regular oscillation pattern of various shapes (taken at 160 frames /sec)



C3. Temperature distribution

In order to check that the oscillation of the drops discussed above is not caused by the non-uniformity of temperature of the hotplate, we have used an infra-red camera to measure the temperature distribution of the hotplate as well as that of the drops. The infrared images show that the temperature distribution of the hotplate is quite uniform. However, the temperature distribution of the drop is not. It seems that the temperatures at the lobes of the drop are higher than that of the center. In addition, the vapor layer can be observed between the droplet and the hotplate. (Fig. 8)

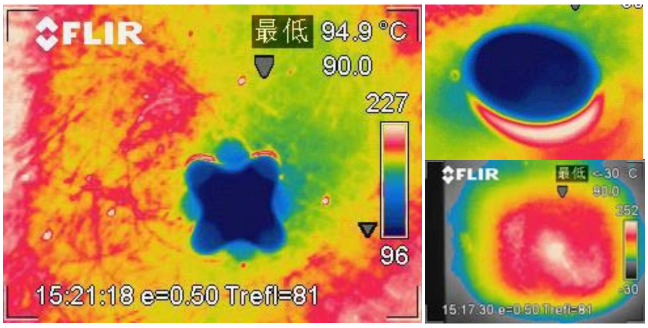


Figure 8. Temperature distribution of the droplet and plate measured by the infra-red camera shown as pseudo color.

C4. Lifetime vs. Temperature

The relationship between the temperature and the lifetime of the droplet on a heated plate is measured via the following procedure:

1. We added one drop at each controlled temperature and recorded the lifetime of the droplets as the time they survive on the plate in seconds.(Fig. 9)

2. We separately added 5, 10, 15 and 20 drops on the plate at different temperatures as 350℃、400℃、450℃ and recorded their lifetime and to investigate the relationship between the volume of the droplets and the temperature of the plate.(Fig. 10)

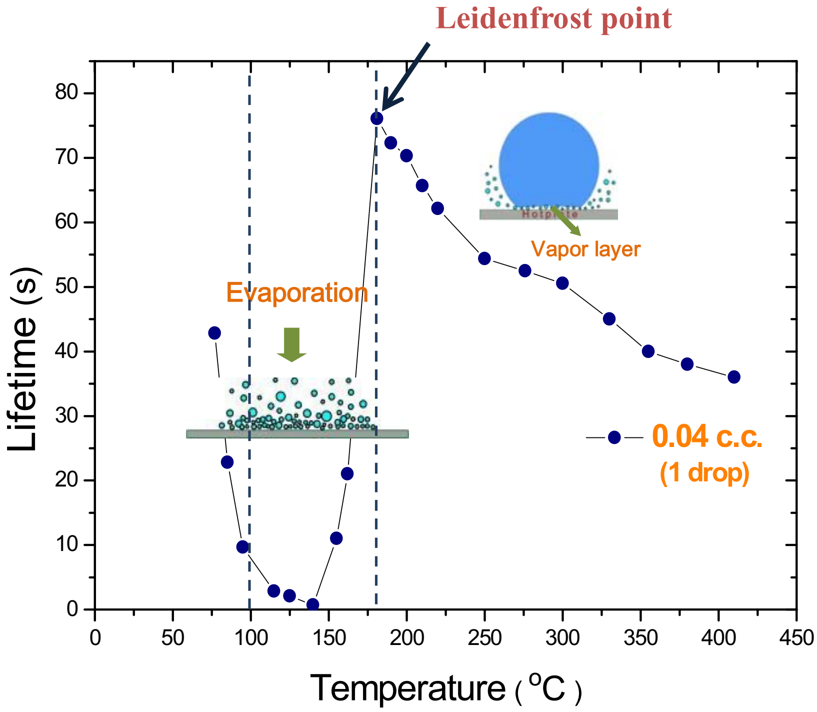


Figure 9. The lifetime of one droplet as a function of the temperature of the hotplate.

Figure 9 shows the results of measured lifetime as a function of temperature. With the help of the thermal couple, we find that when the temperature of the heating plate goes up, the lifetime of the water due to evaporation does not always decrease as one would intuitively think. Instead, when the temperature of the hotplate is higher than certain critical temperature, the lifetime of the droplet suddenly increases. This critical temperature is called the “Leidenfrost point”. When the temperature is above the Leidenfrost point, a vapor layer is formed between the droplet and the hotplate; insulating the heat transfer from the hot plate to the water drop. Thus the water can survive much longer. (Fig. 9)

For the effect of drop volume on lifetime, our result is shown in Figure 10. We obtained that the bigger droplet the longer is its lifetime on the plate. However, the equimultiple volume of the droplet does not mean that the equimultiple increasing lifetime of the droplet. As one can see in Fig. 10, at the same temperature of the multiple drops, the lifetime does not increase in multiple. That tells us one thing. The height of the droplet changes so that the contact area will not increase in multiple ways. The lifetime of the droplet is proportional to the contact area of the hotplate.

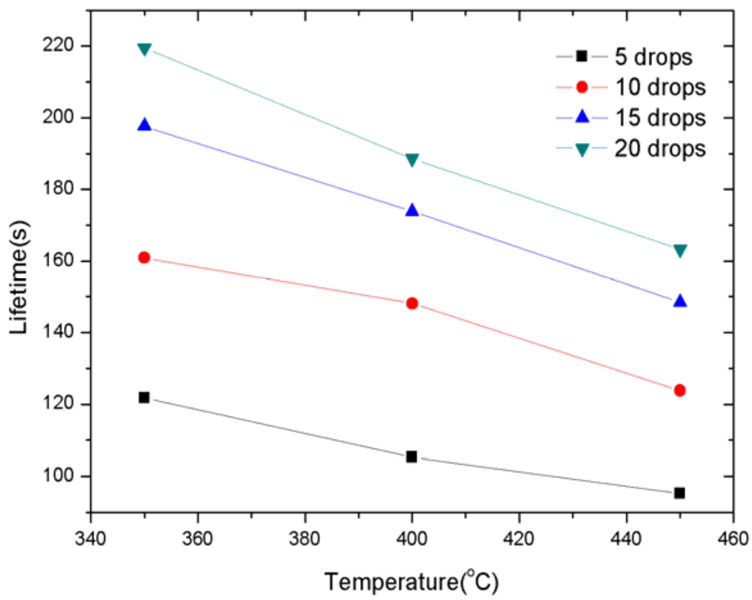


Figure 10. The lifetime of different volume of droplet as a function of the temperature of the hotplate.

C5.Temperature vs. Number of lobes

The relationship between the temperature of the plate and the number of lobes were investigated by the following steps:

1) We controlled the temperature of the plate at 250℃, 275℃, 300℃, 325℃, 350℃, 375℃, 400℃, 425℃, 450℃ and 475℃.

2) Drops of different size were added to the hotplate controlled at these temperatures for several times, and the largest number of lobes formed by the droplets at each temperature was recorded.

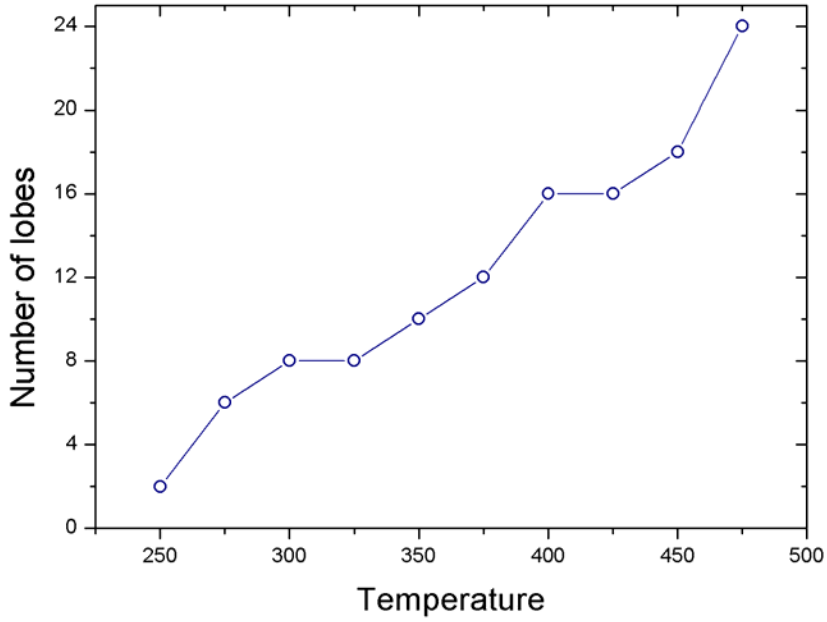


Figure 11. The number of lobes as a function of the temperature of the plate.

As can be seen in Fig. 11, the number of lobes is proportional to the temperature. As the temperature of the plate increases, the maximum number of lobes also increases.

The averaged observed radius of the drop as a function of lobes is plotted in Figure 12. Note that the error bars are the standard deviations of the observed radii for a fixed number of lobes.

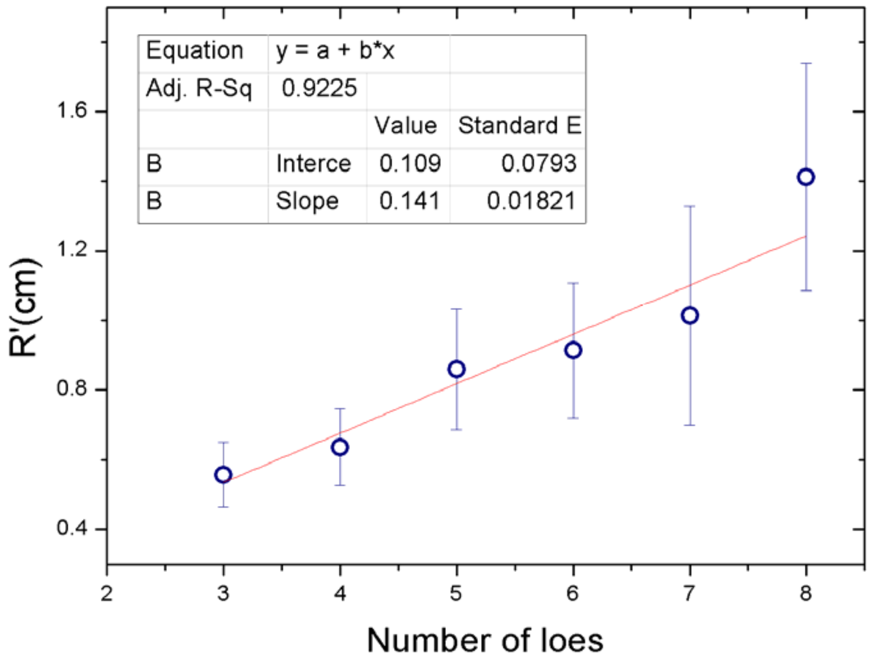


Figure 12. The average radius of different number of lobes at different temperatures.

It can be clearly seen from Fig. 12 that the radius of the drop is proportional to the number of lobes. That means the most important factor which determines the shapes of a vibrating drop is the size of the drop.

C6. Temperature vs. Pressure of the vapor layer

We first observed that fort a given temperature of the hotplate, there is a maximum volume above which no stable drops can be formed on the hotplate. When the volume of the drop exceeds this maximum size, the vapor layer cannot lift the drop against gravity so that the drop will be in direct contact of the hotplate and thus evaporate very fast. To further study the properties of this thin vapor layer, we implemented the following experiments.

We first investigated the maximum radius of the drop as a function of temperature of the hotplate. The result is shown in Fig.13.

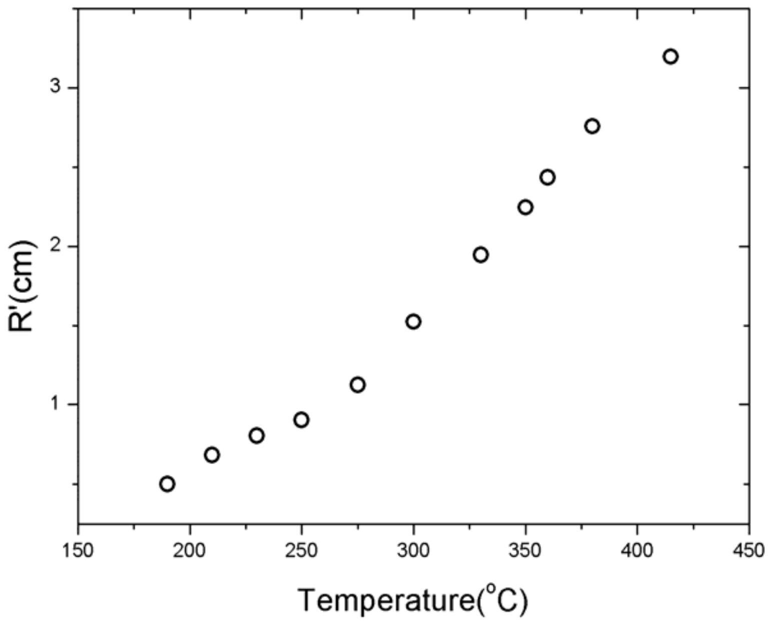


Figure 13. The maximum radius as a function of the temperature of the hotplate.

The measured maximum R' is related to the height of the drop if the volume of the droplet is known. To calibrate the height of the drop as a function of measured R', the following experiment has been performed:

1) We smeared a thin layer of wax on the plate at room temperature to stimulate the contact angle of the drop on the hotplate.
2) Known amount of water is added the hotplate to form drops with different size.
3) Heights of the drops can then be determined by the measured R' and the known volume added.

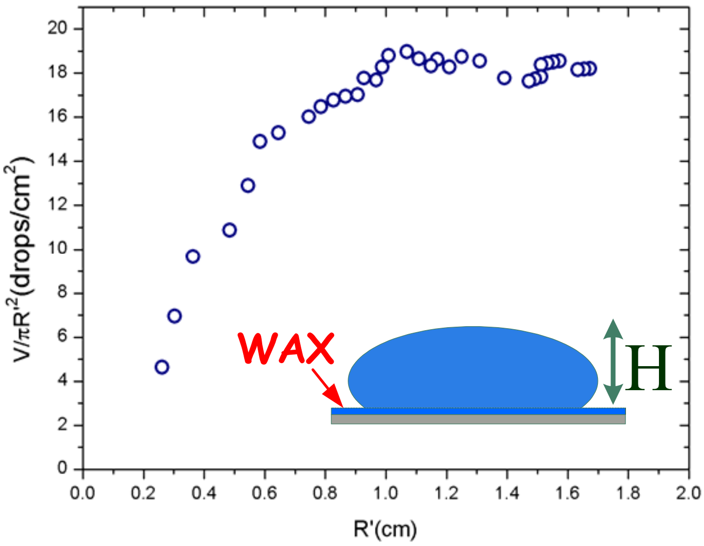


Figure 14. The calibration curve of the height of the droplet as a function of the measured radius.

Using the calibration curve (Fig. 14), we can re-plot the result of Figure 13 in terms of height as Figure 15. (a) .

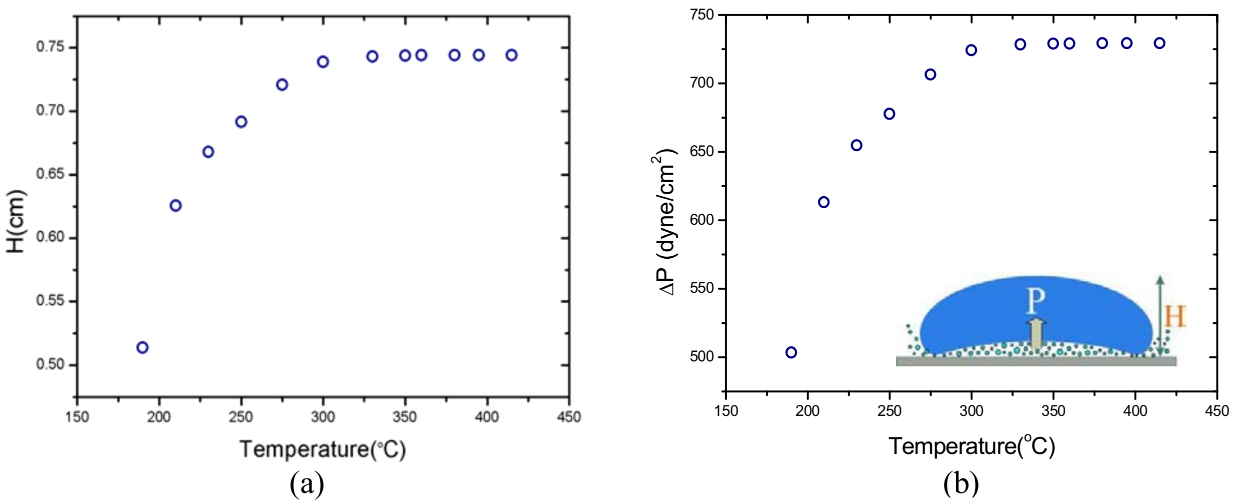


Figure 15. (a) The height of the droplet as a function of the temperature of the hotplate.
(b) The pressure of the vapor layer as a function of the temperature of the hotplate.

From the height (Fig. 15(a)), we can calculate the pressure difference in the vapor layer by using the equation of $\Delta P = \rho g H$ (Fig. 15(b)). In this figure, the pressure of the vapor layer and also the height approaches a constant beyond 300℃, because the height of the water has reached its maximum that can be supported by the surface tension of the water.

C11. Radius vs. Frequency

When considering the vibration of a sphere in its natural frequencies, Rayleigh had already developed an equation to describe the relationship among the frequency, the number of lobes and the radius as:

$$\frac{\omega^2}{n(n-1)(n+2)} = \frac{\sigma}{\rho} \times \frac{1}{R^3}$$

where σ is the surface tension of water, ρ is the density of water and n is the number of lobes in the pattern.

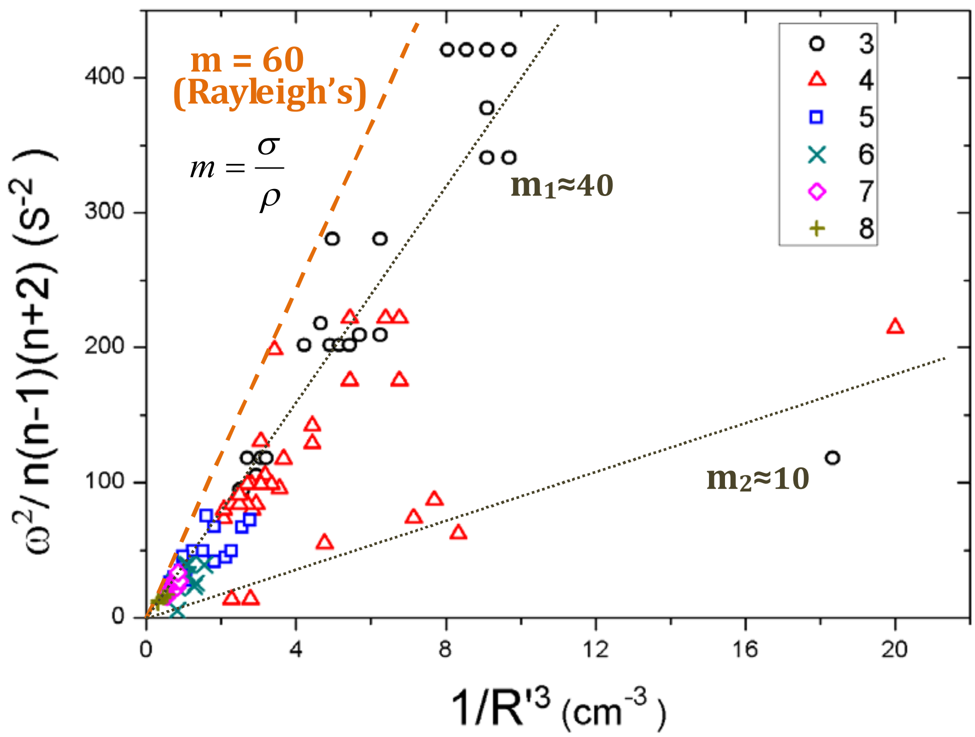


Figure 16. Experimental data plotted in the form of the Rayleigh's relation mentioned above.

In Figure 16, we have used Rayleigh's form to analyze our experimental results. In Rayleigh's equation, $\omega^2/n(n-1)(n+2)$ is proportional to $1/R^3$. Therefore the slope in the Figure is equal to σ/ρ. So in Rayleigh's prediction, the orange slope is 60 (cgs). But as it can be seen from the figure, there seems to be two lines formed by the data. One line is close to Rayleigh's prediction but not quite the same. We are not surprised by this result. In Rayleigh's case, R used is in the radius of a sphere. But in our case, the droplet is more like a Frisbee. So the radii used in the Figure are R' (Figure 16) which is different from Rayleigh's R. In Figure 16 different colors are used in the figure for droplets of different shapes (n). It can be seen that they are ordered by their radii. That means when the radius of the droplet increases, the number of the lobes also increases.

Finally, let us look at the other line which is not close to the prediction of Rayleigh. This line forms a slope (m_2) of about 10 which is about 4 times smaller than m_1. This means that some of the droplets, with the same conditions such as the number of lobes and the radius, can oscillate in two different ways such that the frequencies of the two oscillating modes are related by a factor of two.

D. Discussions

In this section, we will discuss our findings by asking ourselves a couple of questions and providing them with some possible answers.

D1. How does a droplet sustain its oscillation?

There is a paper [1] that studied the motion of drops on a vibrating Teflon plate at room temperature. Both the vibrating plate and a heated plate create the same phenomena of the oscillating drops. However, in Ref [1], they used mechanical forces to oscillate the drops but in our case, we only heated the drop. It can be seen that the heat transferred into the drops creates fluctuations on the surface due to local heating. Presumably, these surface energies are collected into the normal modes (natural frequencies) of vibrations of the droplet.

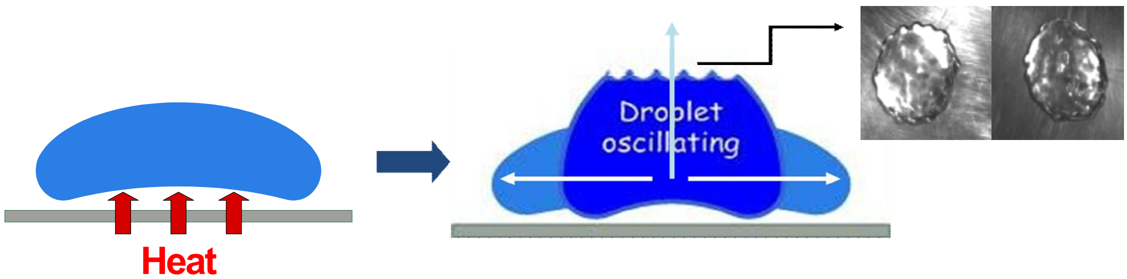


Figure 17. The formation of the fluctuations on the surface of the droplet.

D2. Why are there two lines (slopes) in Fig. 16 ?

These two lines are vibration in different way in droplets of the same condition such as the radius and number of lobes. The period of vibration is doubled in one of the modes. This phenomenon can be observed more easily for smaller droplets. We have observed that while the droplets getting smaller by evaporation, their motions are easier to be influenced by the thermal instability and can become less regular or chaotic. It is known that one of the processes for a periodic system to become chaotic is through the route of "period doubling". It is possible that our system is getting chaotic by "period doubling" as the droplets are getting smaller.

D3. What can we learn from the dependence of ΔP on temperature?

The ΔP is created by the evaporation of water from the drop into vapor and this vapor will support the weight of the droplet while it escapes. This process can be described by using the lubrication theory to determine the thickness (e) of the vapor layer [3]. In this model, a steady state is formed by the associating the heat transfer rate with the vapor escape rate.

$$\frac{dm}{dt} = \frac{\kappa}{L}\frac{\Delta T}{e}\pi R^2 = \rho_v \frac{2\pi e^3}{3\eta}\Delta P$$

Evaporation rate — Heat transfer — Flow in the vapor layer

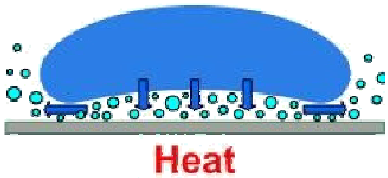


Figure 18. The formation of the vapor layer and the lubrication flow during escape.

This leads to the determination of the vapor layer thickness, as a function of the given Temperature as:

$$e = \left(\frac{3\kappa\Delta T\eta}{2L\rho_v\Delta P}\right)^{\frac{1}{4}} R^{\frac{1}{2}}$$

κ = 0.032W/m/k : thermal conductivity of the vapor layer.
e : thickness of the vapor layer
L = 2.26×10^6 J/kg : latent heat of evaporation
ΔP : pressure imposed by the drop.
η = 1.63×10^{-5} Pa s : vapor viscosity
ρ_v = 0.5 kg/m^3 : vapor density
ρ : liquid density
ΔT /e : temperature gradient

Figure 19 is the result of the measured thickness as a function of temperature from Figure 15(b). The linear relationship clearly reveals that a higher temperature can produce a thicker vapor layer, so as to support a large amount of water, as previously observed.

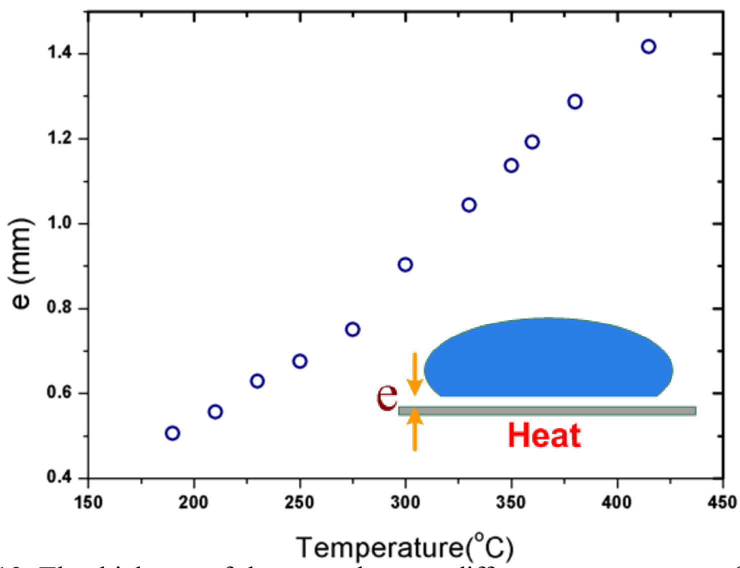


Figure19. The thickness of the vapor layer at different temperatures of the hotplate.

D4. Why does higher n occur only for large R’?

As mentioned above (Figure 6), the characteristic size of the lobes is more or less the same. Therefore, the number of lobes can only be increased for larger drops.

E. Conclusions

1. The dancing water droplet does not evaporate immediately because there is a vapor layer beneath it.
2. The higher the temperature the thicker the vapor layer is, being able to support larger water droplets.
3. The larger the size the more lobes it contains. The lobes have a characteristic size consistent with the capillary length.
4. The dancing motion of water droplets consists of oscillations and rotations.
5. The oscillation is self-driven and operates in normal modes. This oscillation is consistent with Rayleigh’s equation.
6. The observed dual frequencies for the same size and shape may be due to the “period doubling” route to chaos.

F. Future work

In the above discussions, we have concentrated our observation on oscillations. However, for some conditions, there are rotations of the droplets as shown in Figure 18. We find that the rotation of the droplets is not caused by the asymmetry of the plate as the rotation can be both clockwise and anticlockwise. Presumably, this rotation is the result of some self-induced instability of the system. We would like to investigate this self-induced rotation of the droplet in the future.

Figure 20. The observation of rotation of droplets on the hotplate. Both clockwise and anti-clockwise rotations can be observed as shown.

G. Acknowledgement

I would like to thanks Prof. C. K. Chan of Academia Sinica and Prof. Jiun-Huei Proty Wu of National Taiwan University for the useful discussions during the preparation of this paper. Also the author is thankful to her families for their encouragement and support.

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