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科　　　別：物理科

作 品 名 稱：利用浮沈子測量液體表面張力並演示
"Cheerios effect"

得 獎 獎 項：第二名
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我是高頌凱，是嘉義高中二年級生。

小時候，媽媽常帶我去台中科學博物館，就差不多那段時期，我瘋狂的愛上科學，尤其是生物和物理。直到現在，物理那樸素、簡單又明確的特性，仍是我對物理樂此不疲的原因。感謝李文堂老師及林芳妃老師的指導，讓我有這個機會參加從未接觸過的科展，也希望透這次經驗，能使我學到更多校園沒有的事物。

利用浮沉子測量液體表面張力
並演示”Cheerios effect”

摘要

密閉容器置入待測液，放入浮沉子，施加壓力，當浮沉子恰要沒入液中瞬間，因表面張力的總力達極大值且向上，外加壓力(p_1)爲極大值，浮沉子沒入液中；液面減壓，當浮沉子在液面正下方時，外加壓力 p_2，量 p_1、(p_1- p_2)，浮沉子的質量 m，外半徑 R，及玻璃管的體積V_G，可求得液體表面張力。

液面再減壓，浮沉子恰要露出液面時，表面張力的總力達極大值且向下，外加壓力(p_3)爲極小值，量 p_3、(p_2- p_3)，浮沈子的質量 m，外半徑 R 及V_G，應亦可求得表面張力；但實驗時浮沉子漂移到容器邊，並吸附在器壁上，因此發現浮沉子的”Cheerios effect”。

利用浮沉子和容器的相吸及相斥現象,可解釋西式早餐的小穀片放入牛奶中爲何會漂移到碗緣，並支持 Vella 在 2005 年 9 月份美國物理期刊(AJP)認爲 Cheerios effect 的成因除了由於接觸角不同外，浮力、重力、表面張力共同作用，使小穀片間有相吸、相斥現象。

Using a Cartesian diver to measure the surface tension of liquids and demonstrate the "Cheerios effect"

ABSTRACT

The experiment apparatus is equipped with a Cartesian diver by using a glass tube with air trapped inside that floats or submerses in a closed vessel containing liquid.

The external pressure may be varied with a syringe and measured with a water manometer. The maximum pressure P_1 inside the vessel is measured when the diver is just about to sink, where the surface tension that acts on the diver is upward. Then the pressure P_2 of the vessel is measured when the diver is just beneath the liquid surface, where no surface tension acts on the diver. Finally, the surface tension is calculated from P_1, P_2 and the radius of the diver, R.

When the pressure inside the vessel is decreased, the diver will rise. As the diver is about to emerge from the liquid, we get the minimum pressure P_3 inside the vessel, and the surface tension that acts on the diver is downward. By measuring P_3, P_2, and R, the magnitude of surface tension is found to be the same as above.

When the diver is just about to sink into the liquid, it floats to the center of the vessel. When the diver is about to emerge from the liquid, it sticks to the wall of the vessel. This phenomenon is named the "Cheerios effect." Our results again strongly support that the cause of the effect is due to the different contact angles between the diver and water, as well as the balance of gravity and surface tension in the case of the sinking diver, and the balance of buoyancy and surface tension in the case of rising diver as Vella claimed in his paper (AJP **73**, 817 (2005)).

利用浮沉子測量液體表面張力並演示”Cheerios effect”

一、引言：

1. 手施力壓裝有浮沉子的塑膠瓶，浮沉子快要下沈瞬間，和恰在液面正下方時，手施的壓力相差很多，利用此壓力差可準確的測量液體表面張力。
2. “表面物理化學“（註 1）介紹測量表面張力的方法上百種，未見有利用浮沉子測量者。
3. 歷屆全國科展，有兩件第一名作品（註 2、3）利用毛細管爲測量表面工具；毛細管半徑小，若待測液被污染，誤差甚大。本作品將浮沉子置於密閉容器中，利用外加壓力測量液體表面張力：（1）效果非常顯著；（2）準確度極高；（3）待測液不易被污染；（4）原理簡單，不必做繁雜的修正。
4. 2002 年 Güémez [4] 利用浮沉子解釋”Fold catastrophe”現象，在驗證波以耳定律時，未考慮表面張力，所以誤差甚大。我們考慮表面張力的因素之後，得到的實驗結果和理論相吻合。
5. 2005 年 Dominc Vella [5] 提出”The Cheerios effect” 在我們利用浮沉子測量液體表面張力過程中發現浮沉子亦會呈現相同的現象。證明 The Cheerios effect 的成因除了由於接觸角不同外，浮力、重力、表面張力共同作用，使小穀片間有相吸、相斥現象。

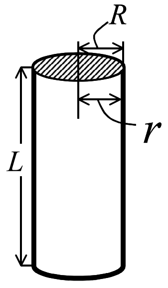
二、研究目的：

1.推導出利用浮沉子測量液體表面張力的公式。

2.利用浮沉子測量液體的表面張力。

3. 利用浮沉子演示說明 The Cheerios effect。

三、原理：



圖一：玻璃管浮沉子

1. 一端封閉的玻璃管，外半徑 R，內半徑 r，質量 m，

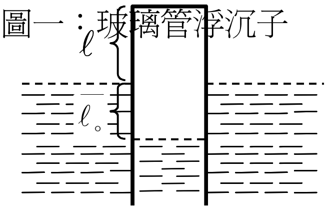
玻璃的體積=$\pi(R^2-r^2)L+\pi r^2 h$，h：底部玻璃厚度。

2. 玻璃管注入適當的水，倒插入盛水的燒瓶中，

部分在空氣中。如圖二(a)所示，沒在液中空氣柱長 ℓ_0 露出液面長度 ℓ，管內部截面積

$A=\pi r^2$，由亞基米德原理：浮體浮力 ＝

浮體重 ＝ 排開液重。



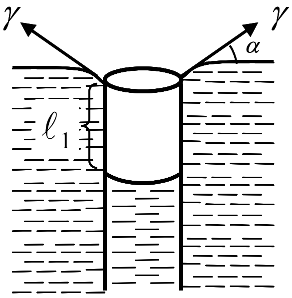
圖二(a)：浮沉子部分在空氣中

$$\therefore\ B = mg = l_0 A\rho g + \pi(R^2 - r^2)(L - \ell)\rho g \quad \cdots \quad (1)$$

ρ ：液體密度

3. 燒瓶封閉，施加壓力至浮沉子快要沒入液中時，液體表面張力的總力達極大值，圖二(b)中的α=90°。外加壓力 P_1，玻璃管沒在液中的體積=$\pi(R^2 - r^2)L = V_G$ 。依帕斯卡原理，管內氣體壓力P_1 。靜力平衡，浮力及表面張力向上，重力向下

$$\Rightarrow \quad V_G\rho g + l_1 A\rho g + 2\pi R\gamma = mg \quad \cdots \quad (2)$$



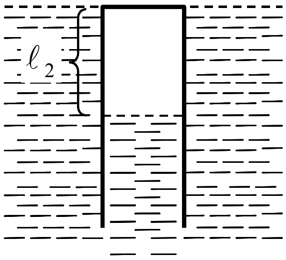
圖二(b)：浮沉子表面張力的作用

4.再略加大壓力，浮沉子沒入液中，減壓使其在液面正下方時，管中氣體壓力P_2 ，$P_1 > P_2$ ，空氣柱長$\ell_1<\ell_2$ ，浮沉子受重力向下，浮力向上而平衡

$$\Rightarrow \quad V_G\rho g + l_2 A\rho \mathrm{g} = mg \quad \cdots \quad (3)$$

由波以耳定律

$$\Rightarrow \ P_0\left(l + l_0\right)A = P_1 l_1 A = \mathrm{P}_2 l_2 \mathrm{A} \quad \cdots \quad (4)$$



圖二(c)：浮沉子在液面下方

5.燒瓶內的氣體再減壓，至浮沉子恰要露出液面的瞬間，表面張力的總力向下且最大，重力向下，浮力向上而平衡，空氣柱長$l_3 > l_1$

$$V_G\rho g + l_3 A\rho g = mg + 2\pi R\gamma \quad \cdots \quad (5)$$

波以耳定律 $P_3 l_3 A = P_2 l_2 A \quad \cdots \quad (6)$

6.由原理 3 和 4 結合公式 (2)、(3)、(4)可求出表面張力：

$$\gamma = \frac{(P_1 - P_2)(m - V_G\rho)g}{2\pi R P_1} \quad \cdots \quad (7)$$

量浮沉子質量 m，有效半徑 R，V_G並由開管壓力計(及氣壓計)量得 P_1 ，P_2，並求出 P_1 - P_2代入(7)可得γ 。

7.另由原理 4、5 結合(3)、(5)、(6) 得表面張力

$$\gamma = \frac{(P_2 - P_3)(m - V_G\rho)g}{2\pi R P_3} \quad \cdots \quad (8)$$

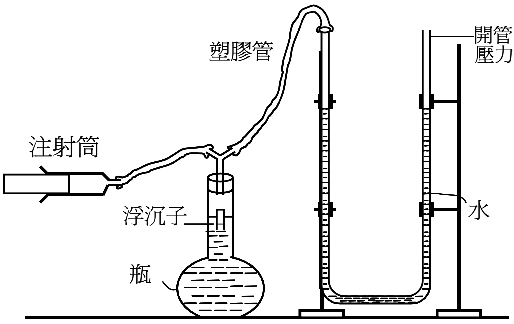
量 m、R、V_G、P_3及 P_2-P_3亦可得γ。

四、裝置：

1.底部平滑的均勻玻璃管，質量 m = 4.58 g，外半徑 R = 0.5 ㎝，內半徑 = 0.4 ㎝， 厚 1 ㎜，長 7 ㎝，做爲浮沉子，用來測量水及甘油的表面張力。另有質量 m =3.37 g，外半徑 R = 0.63 ㎝，內半徑 = 0.58 ㎝， 厚 0.5 ㎜，長 7 ㎝，做爲浮沉子，用來測量酒精的表面張力。

2.燒瓶、燒杯、量筒、天平、橡皮管、三叉管、游標尺、尺、蒸餾水、酒精、甘油、溫度計。

3.自製開管壓力計：120cm 的玻璃管兩隻，用塑膠管連接，下置透明尺及平面鏡，固定在鉛直豎立的木板上而成。管內放入水，測量時眼睛正視水面、尺的刻度、鏡中像成一直線、可準確量出水面高度。



圖三：實驗裝置

五、實驗步驟：

實驗(一) ： 測量玻璃管浮沉子的密度及盛水量

4. 利用游標尺量外徑 R，內半徑 r，長度 L，算出玻璃管玻璃部份的體積V_G，用電子天平測出質量 m。

5. 由公式 (3) $mg = l_2 A\rho g \Rightarrow \ell_2 = \frac{m}{A\rho} = \frac{m}{\pi r^2 \rho}$ 算出空氣柱長ℓ_2，即注入水柱 L-l_2 時，浮沉子恰沒入水中，用注射筒抽掉少許的水，使空氣柱長＞ℓ_2，則可浮在水面。

實驗(二) ：利用原理 3、4，即公式 (7) 測表面張力

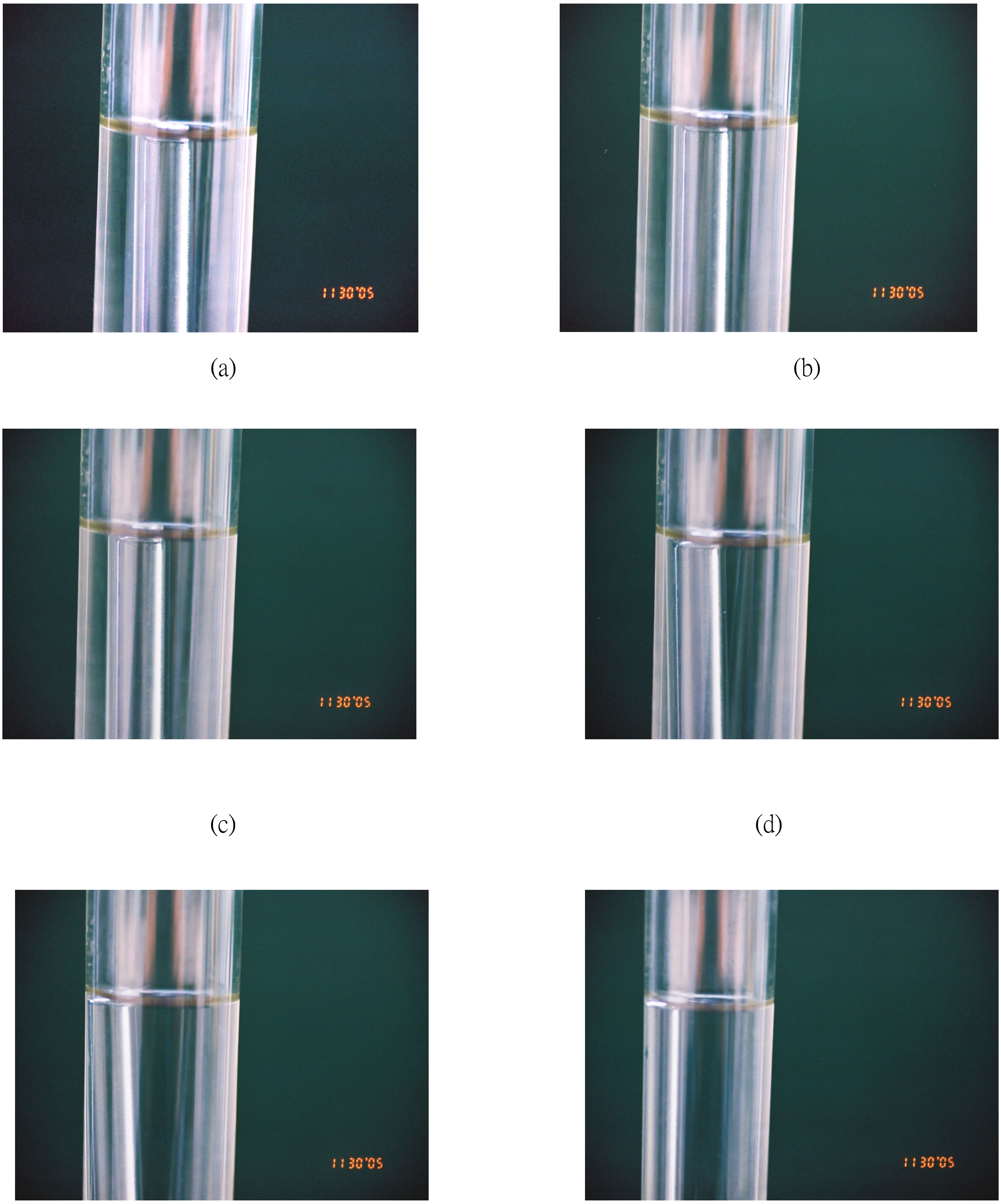
1. 燒杯中放入 t℃的水，玻璃管注入適量的水，倒插在燒杯的水中，調整管中的水位，至約 1 ㎜露出水面，取出玻璃管用塑膠貼紙封住管口，玻璃管用蒸餾水洗乾淨，用夾子夾著倒插入燒瓶中，用夾子撕去貼紙，即形成浮沉子。
2. 記下大氣壓力 P_0，開始用注射筒加壓，至浮沉子快要下沉，即最大壓力 P_1。
3. 注射筒減壓至浮沉子觸及液面，記下 P_2。
4. 量 P_1、P_2、R、m、V_G 代入 (7) 求出 γ 。


圖四-1 浮沈子快要下沈，浮沉子旁邊的水面呈凸透鏡式彎月形


圖四-2 浮沈子恰要下沈，浮沉子在容器的正中央 (Cheerios effect 的相斥現象)

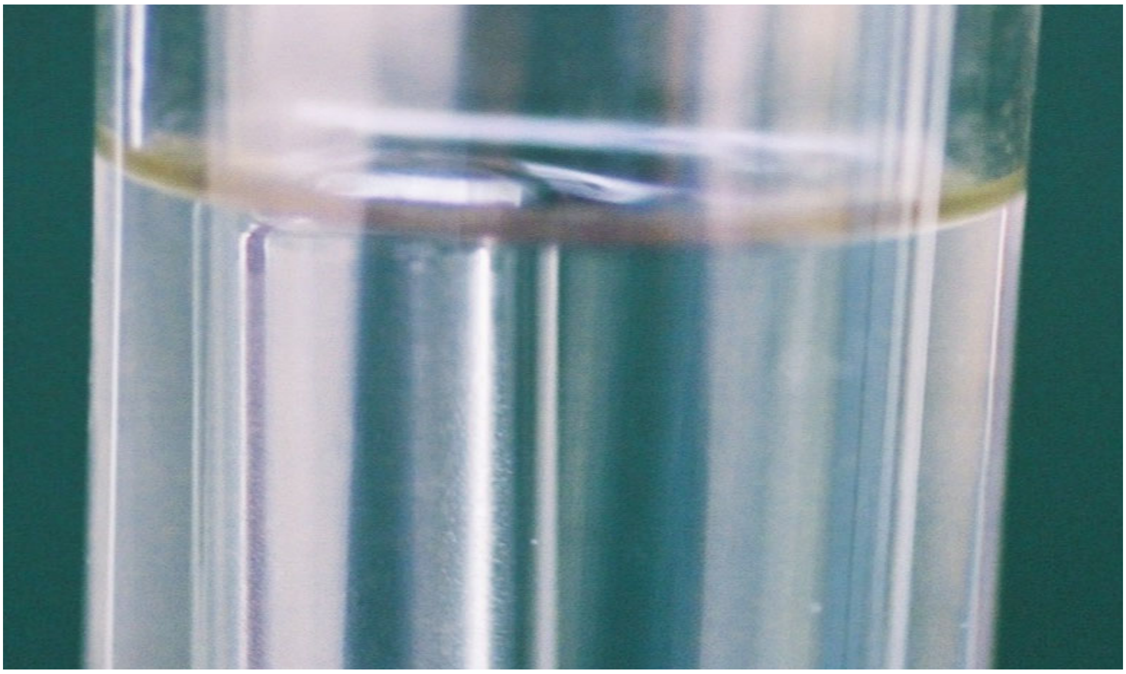
實驗(三) ：利用原理 4、5 即公式 (8) 測 γ

1. 先施加壓力使浮沉子沒入水中，緩緩減壓至浮沉子頂端恰觸及液面，量得壓力 P_2。再減壓至浮沉子恰要脫離液面，量 P_3。
2. 由 (8) ⇒ $\gamma = \dfrac{(P_2 - P_3)(m - V_G \rho)g}{2\pi R P_3}$ 求出 γ 。



(a) (b) (c) (d) (e) (f)

圖五：浮沉子由(a)至(f)從容器中央吸到左端



圖六：Cheerios effect 的相吸作用，浮沉子右上方水的表面張力方向和水平成 θ = 35° ；左方成水平，所以，浮沉子被吸（拉）向左方

六、實驗結果：

1.浮沉子長 7 ㎝，內半徑 0.4 ㎝，外半徑 0.5 ㎝，由圖四可量得有效半徑=0.40cm。

2.質量 m = 4.58g 的玻璃管，由此算出玻璃密度 ρ_G = m/V = 2.26g/cm^3。

3. 表一： 利用公式 (7) 求水的表面張力(水溫 25℃)

次數	1	2	3	4	5
P_1(cm-H_2O)	1156.4	1148.3	1152.6	1157.0	1153.7
P_2(cm-H_2O)	1074.1	1066.2	1070.9	1074.2	1070.8
γ (dy/cm)	71.9	72.2	71.6	72.3	72.6

γ = (72.1 ± 0.2)dy/cm

4. 表二： 利用公式 (7) 求酒精的表面張力 (溫度 25℃)

次數	1	2	3	4	5
P_3(cm-H_2O)	1095.8	1101.2	1099.3	1093.7	1098.9
P_3(cm-H_2O)	1059.1	1063.4	1062.9	1056.0	924.8
γ (dy/cm)	21.9	22.5	21.7	22.6	22.0

$\gamma = (22.1 \pm 0.2)$dy/cm

5. 表三： 利用公式 (7) 求甘油的表面張力 (溫度 25℃)

次數	1	2	3	4	5
P_2(cm-H_2O)	1143.2	1145.2	1146.7	1141.8	1144.2
P_3(cm-H_2O)	1054.8	1055.4	1057.3	1053.8	1055.6
γ (dy/cm)	62.3	63.2	62.8	62.1	62.4

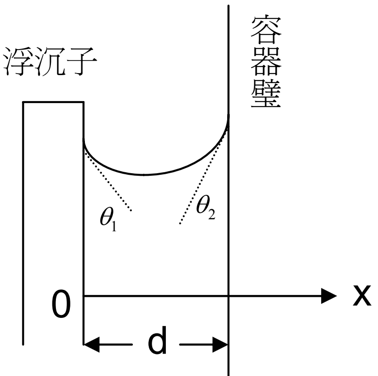
$\gamma = (62.6 \pm 0.2)$dy/cm

6.原來要用公式(8)測表面張力，來和利用公式(7)測量的結果做比較，但當浮沉子由水中快要露出水面時，浮沉子旁邊的水面形成凹透鏡式的彎月形，浮沉子漂移向容器壁，吸附在壁上，施甚大的負壓才會脫離液面，測得結果不理想，探究其原因，發現了浮沉子的”Cheerios effect”。

七、對”Cheerios effect”的解釋：

1.西式早餐的穀片(cereals, Cheerios 爲穀片的廠牌名)，放入裝有牛奶的碗內時，會聚成一團且漂浮到碗緣，J. Walker 在 The flying circus of physics 書中有 Two attractive cheerios 一節，對此現象的解釋爲：牛奶和碗間的接觸角和穀片與牛奶間的接觸角不同產生的效應。Vella(2005)利用一圖釘及其塑膠帽的相吸及相斥，證明此效應並非只是接觸角不同引起的；他並同時推導兩個小圓圈相吸互相靠近的距離對時間公式。

2.玻璃管浮沉子及玻璃容器和水間的接觸角一定（跡近零度），浮沉子快要從水面露出時被吸到容器邊緣，空氣液體界面因器壁的存在，形成凹透鏡形的彎月形，施加的負壓使浮沉子受浮力向上，因彎月形作用，只好順著彎月形向上方漂浮，看起來好像被器壁吸引。



圖七：浮沉子和容器壁
的作用力

3.圖七所示浮沉子和容器壁以及液體間的接觸角各爲θ_1, θ_2，液面由於表面張力作用，比水平面高出 h，$\gamma\frac{d^2h}{dx^2} = \rho gh \cdots(9)$

微積分處理（見附錄）後，得到浮沉子受到器壁單位長度的作用力F_h，

$$F_h = -\frac{\gamma}{2}(\cot^2\theta_1 + \cot^2\theta_2 + 2\cot\theta_1\cot\theta_2\cosh\lambda d)\ \cdots(10)$$

d 爲浮沉子到器壁的距離，$\lambda = \sqrt{\frac{\rho g}{\gamma}}$，(10)式中負號表示相吸。如圖五所示液體均呈彎月形向上即$\cot\theta_1\cot\theta_2 > 0$，所以浮沉子被吸向器壁。若要相斥，換言之，$F_h$要爲正値，必要條件爲$\cot\theta_1\cot\theta_2$必須小於零。圖四所示，浮沉子受到最大外加壓力快下沉時，玻璃管旁的水形成凸透鏡式的彎月形$\cot\theta_1\cot\theta_2 < 0$，浮沉子被推到容器中央，好像受到斥力一樣。

4.在密閉容器中施壓力使浮沉子漂移到容器正中央，緩緩降壓，至壓力等於大氣壓力時，浮沉子開始向器壁移動，如圖五所示。利用攝影機拍下浮沉子移動的軌跡，再定格量出位一對時間關係，其數學式値得進一步研究。

八、討論：

1. 測量數據中，測量值都是三位或三位以上的有效數字，我們採用三位有效數字。測得 25℃的水、酒精、甘油的表面張力分別爲 72.1dy/cm、22.1dy/cm、62.6dy/cm，和公認值的 71.97dy/cm、22.27dy/cm、63.28dy/cm 誤差分別爲 0.18%、0.76%、1.07%。

2.測量表面張力的方法有許多種，全國科展第十八屆及第三十五屆物理科第一名的作品，都是利用毛細管爲測量工具。我們利用浮沉子來測量，截面半徑比毛細管大許多，所以表面張力的現象更顯著。

3..本實驗僅需測量浮沉子玻璃管的質量(m)，外半徑(R)，玻璃的體積以及用開管壓力計量 P_1、P_2、P_3 以及壓力的變化 ΔP，極顯著且易量，而且測量過程中，浮沉子均在密閉容器內，不易受外界污染，所以只要在放入浮沉子之前，將浮沉子和燒瓶洗乾淨，用夾子置入浮沉子後，外界對測量系統幾乎沒有影響。

4..本實驗用很簡單的器材作精密的測量實驗。做完本實驗使我們對(1) 帕斯卡原理 (2)亞基米德原理(3)波以耳定律(4)表面張力的測量能更進一步的了解。

5.利用本方法測表面張力時，要利用像玻璃管體積不受壓力改變的管子來作浮沉子，不可用像些塑膠滴管之類的易形變的管子，以防加壓後管子的容積改變。

6.浮沉子在密閉容器中快下沉時，表面張力向上，使管子受斥漂移至容器中央，快由液中浮起時，表面張力向下，使管子受吸力漂移至容器邊緣，利用浮沉子演示並作 Cheerios effect 實驗效果很好。

九、結論：

1.本實驗利用浮沉子測量表面張力，所用原理均爲高中物理課本所述的：表面張力、帕斯卡原理、波以耳定律、亞基米得原理的綜合應用。

2.本實驗測量表面張力時，事先測好玻璃管的質量，外徑，玻璃體積，進行測量時不會干擾、污染到待測液，且溫度容易控制，所以可以用來研究表面張力隨溫度的變化。

3.利用浮沉子來演示 Cheerios effect 不但效果顯著，而且水和玻璃的接觸角固定，浮沉子下沉時和器壁相斥，上浮時相吸，並非僅接觸角的作用，還有重力、表面張力及浮力共同的作用。

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Cartesian Diver, Interfacial Phenomena, and the Cheerios Effect

ABSTRACT

A Cartesian diver is used to study the liquid-solid-air interfacial phenomena. We found that the surface tension of the liquid can be related to the two critical pressures, at which the diver is about to sink and about to emerge. Frequently, the measured surface tension is within 1% that of the well-recognized value, demonstrating that Cartesian diver is a simple, trustworthy, and inexpensive classroom apparatus for surface tension study.

In the critical regime on sinking, a capillary wave, consisting of progressively shrinking rings, may be created. The number of rings was found to increase with increasing sinking depth. We believe that this phenomenon is created by the competition between the solid-liquid interfacial dragging force, the surface tension of the liquid, and the pressure gradient.

Upon sinking by increasing vessel pressure, the diver moves to the center due to the appearance of repulsive forces between the diver and the vessel wall, which drives the system into an equilibrium state, prior to forming the capillary wave. On reducing pressure, the diver moves to the side due to attractive forces created as the contact angle between the diver and the liquid was greater than 90°. These phenomena are known as the Cheerios effect. Apparently, the Cheerios effect can be distinguished into the stable repulsive and attractive phases. We found that it is easy to create the two phases of the Cheerios effect by changing the internal pressure of the vessel.

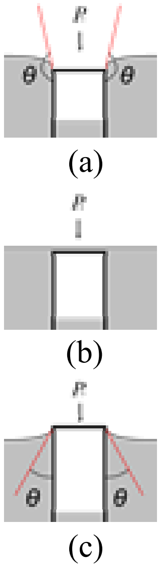
INTRODUCTION

The Cartesian diver, an old science toy, which is placed in a vessel containing water, will sink when the pressure in the vessel is increased, and rise when pressure is decreased. It is always used to illustrate Archimedes's principle, Pascal's principle, and Boyle's law.

Our experiment is equipped with a Cartesian diver made of an acrylic tube with air trapped inside that floats or submerses in a closed vessel containing liquid. We measure the two critical pressures, at which the diver is about to sink and about to emerge. The surface tension is calculated accurately from these two critical pressures and the effective radius of the diver.

In the critical regime on sinking, a capillary wave consisting of progressively shrinking rings may be created. We believe that this phenomenon is created by the competition between the solid-liquid interfacial dragging force, the surface tension of the liquid, and the pressure gradient.

When the diver is just about to sink into the liquid, it floats in the center of the vessel. As the diver is about to emerge from the liquid, it moves toward and finally sticks to the wall of vessel. These phenomena are known as the Cheerios effect. We can control the attraction or repulsion of the Cheerios effect by changing the internal pressure of the vessel.



(a)
(b)
(c)

Fig. 1 The Cartesian diver in liquid

THEORY

1. An acrylic tube of length L, radius R, mass m, density d, V=m/d. The diver not completely filled with air, is inverted in a vessel containing liquid, as shown in Fig.1. (a)

M: The weight of the diver,

$$B = mg = l_0 A\rho g + \pi(R^2 - r^2)(L - \ell)\rho g \ldots (1)$$

ρ: density of liquid.

If we increase the pressure in the vessel, and measure the maximum pressure P_1 when the diver is just about to sink, where the surface tension that acts on the diver is upward, in Fig.2 (b), $\alpha=90^\circ$ we get:

$$V_G \rho g + l_1 A\rho g + 2\pi R\gamma = mg \ldots (2)$$

Then add more pressure and the diver will sink. Now decrease the pressure when the diver is just beneath the liquid surface, as in Fig. 1(c), where no surface tension acts on the diver. In this situation, the pressure of the vessel is P_2.We get $V_G \rho g + l_2 A\rho g = mg \ldots (3)$

By using Boyle's law: $P_0(l + l_0)A = P_1 l_1 A = P_2 l_2 A \ldots (4)$

2. When the pressure inside the vessel is decreased, the diver will rise. As the diver is about to emerge from the liquid, we measure the minimum pressure P_3

inside the vessel. The surface tension that acts on the diver is downward.

$V_G \rho g + l_3 A \rho g = mg + 2\pi \mathrm{R} \gamma$ … (5)

Boyle's law: $P_3 l_3 A = P_2 l_2 A$ … (6)

Combine Equations (2), (3), and (4), we get the equation for measuring the surface tension γ : $\gamma = \dfrac{(P_1 - P_2)(m - V_A \rho)g}{2\pi R P_1}$ … (7)

3. Combine Equations (3), (5), (6), we get another equation for measuring the surface tension γ : $\gamma = \dfrac{(P_2 - P_3)(m - V_G \rho)g}{2\pi R P_3}$ … (8)

4. The equation of the interface is determined by the condition that the pressure change across the interface due to the surface tension is:

$\gamma \dfrac{d^2 h}{dx^2} = \rho g h$ … (9)

Let $h = Ae^{\lambda x} + Be^{-\lambda x}$ … (10)

$h^{'}(x) = \dfrac{dh}{dx} = A\lambda e^{\lambda x} - B\lambda e^{-\lambda x}$ … (11)

$h^{''}(x) = \dfrac{d^2 h}{dx^2} = A\lambda^2 e^{\lambda x} + B\lambda^2 e^{-\lambda x}$ … (12)

Combine Eq. (11) and Eq. (14), we get

$\lambda = \sqrt{\dfrac{\rho g}{\gamma}}$ … (13)

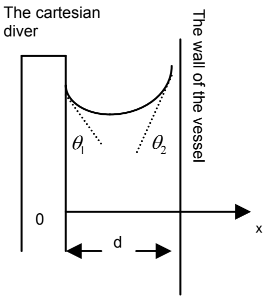


Fig.2. Physical parameters associated with the Cheerios effect

Let the region on the left hand side of the diver be the region between the diver and vessel be region 2.

∵h(∞)=0， $\dfrac{dh}{dx} = \cot\theta_1$

$h_1(x) = A_1 e^{\lambda x} + B_1 e^{-\lambda x}$ ，at x=－∞， $h_1(x) = 0 \Rightarrow B_1 = 0$ ，

$h_1^{'}(x) = A_1 \lambda e^{\lambda x} - B_1 \lambda e^{-\lambda x}$ ， $h^{'}(0) = \cot\theta_1$ we get $A_1 = \dfrac{\cot\theta_1}{\lambda}$

$\therefore h_1(x) = \dfrac{\cot\theta_1}{\lambda} e^{\lambda x} \Rightarrow h_1(0) = \dfrac{\cot\theta_1}{\lambda}$ … (14)

∵at x=0, $\dfrac{dh}{dx} = -\cot_1$ ，at x=d, $\dfrac{dh}{dx} = \cot\theta_2$

$h_2(x) = A_2 e^{\lambda x} + B_2 e^{-\lambda x}$

$h_2^{'}(0) = A_2 \lambda - B_2 \lambda = -\cot\theta_1$ … (15)

$h_2^{'}(d) = A_2 \lambda e^{\lambda d} - B_2 \lambda e^{-\lambda d} = \cot\theta_2$ … (16)

Combine eq. (15) and Eq. (16), we get

$$A_2 = \frac{\cot\theta_2 + e^{-\lambda d}\cot\theta_1}{2\lambda\sinh\lambda d}$$

$$B_2 = \frac{\cot\theta_2 + e^{\lambda d}\cot\theta_1}{2\lambda\sinh\lambda d}$$

$$\therefore h_2(x) = \frac{1}{\lambda}\frac{\cot\theta_1\cosh\lambda(d-x) + \cot\theta_2\cosh\lambda x}{\sinh\lambda d} \ldots (17)$$

The value of the horizontal force per unit length act on the diver, Fh

$$F_h = -\int_{h_1(0)}^{h_2(0)} \rho g h dh = \frac{1}{2}\rho g\left[h_1(0)^2 - h_2(0)^2\right]$$

$$= -\frac{\gamma}{2}\frac{(\cot\theta_1\cosh\lambda d + \cot\theta_2)^2}{\sinh^2\lambda d} + \cot^2\theta_1$$

$$= -\frac{\gamma}{2}\frac{\cot^2\theta_1 + \cot^2\theta_2 + 2\cot\theta_1\cot\theta_2\cosh\lambda d}{\sinh^2\lambda d} \ldots (18)$$

Where the sign convection is such that F_h <0 corresponds to attraction between the diver and vessel. In Eq. (18) the values of: $\cot^2\theta_1$ ， $\cot^2\theta_2$ ， $\sinh^2\lambda d$ ， $\cosh\lambda d$ are all positive. The effect of repulsion can occur only if $\cot\theta_1\cot\theta_2$ <0.

Fig.3. Apparatus employed in this study.

APPARATUS

1. An acrylic tube of length 7cm, radius 0.5cm, mass 4.58g, acrylic volume 2.48 cm3.
2. A cylindrical vessel of radius 5cm, length 30cm, the top of the vessel is removable with screw.
3. A digital manometer.
4. An air supply for increasing or decreasing the pressure in the vessel.

EXPERIMENT

1. A Cartesian diver, not completely filled with distilled water is inverted in a beaker also containing distilled water; and the diver emerges above the water surface about 0.5mm. Then, cover the mouth of the tube with a thin plastic film. Clean the diver with distilled water.
2. Pour distilled water into the vessel, and remain an air column about 2cm high. Take the clean diver with a clip, and put it into the vessel.
3. Close the vessel tightly, and then measure the maximum pressure P_1 when the diver is just about to sink. Add a little pressure, and the diver will sink. Now, decrease the pressure, till the diver is just beneath the water surface, and measure the pressure P_2.
4. By measuring the value of P_1, P_2 and the effective radius, we can calculate the surface tension of water.
5. Replace water with ethyl alcohol and glycerin, and repeat the same process.

RESULTS

1. Table 1. The surface tension of water at 25℃

No.	1	2	3	4	5
P1(cm-H2O)	1156.4	1148.3	1152.6	1157.0	1153.7
P2(cm-H2O)	1074.1	1066.2	1070.9	1074.2	1070.8
γ(dy/cm)	71.9	72.2	71.6	72.3	72.6

γ= (72.1 ± 0.2) dy/cm

2. Table2. The surface tension of ethyl alcohol at 25℃

No.	1	2	3	4	5
P2(cm-H2O)	1095.8	1101.2	1099.3	1093.7	1098.9
P3(cm-H2O)	1059.1	1063.4	1062.9	1056.0	924.8
γ(dy/cm)	21.9	22.5	21.7	22.6	22.0

γ= (22.1 ± 0.2) dy/cm

3. Table3. The surface tension of glycerin at 25℃

No.	1	2	3	4	5
P2(cm-H2O)	1143.2	1145.2	1146.7	1141.8	1144.2
P3(cm-H2O)	1054.8	1055.4	1057.3	1053.8	1055.6
γ(dy/cm)	62.3	63.2	62.8	62.1	62.4

γ= (62.6 ± 0.2) dy/cm


Fig.4 Repulsion of the Cheerios effect


Fig.5 Progressive shrinking rings

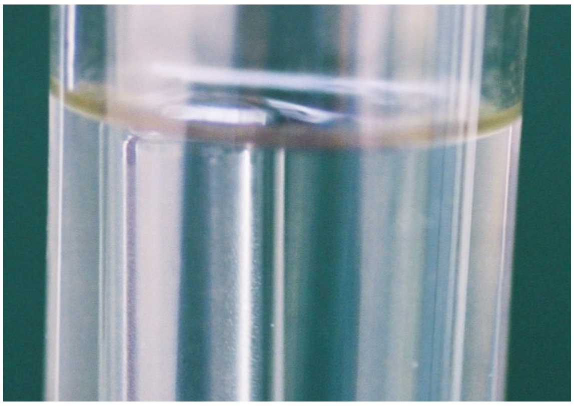


Fig.6 Attraction of the Cheerios effect

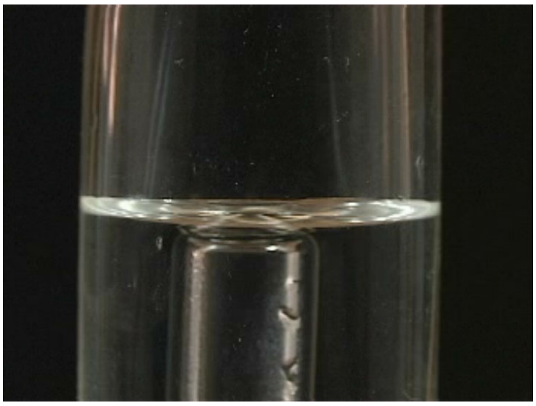


Fig. 7(a)

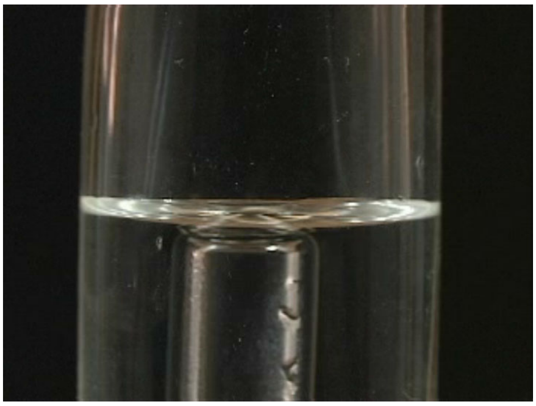


Fig. 7(b)

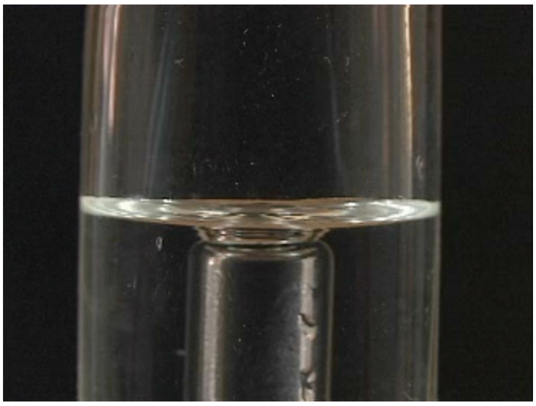


Fig. 7(c)

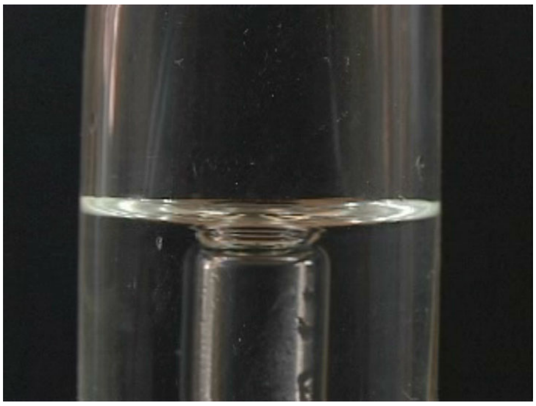


Fig. 7(d)

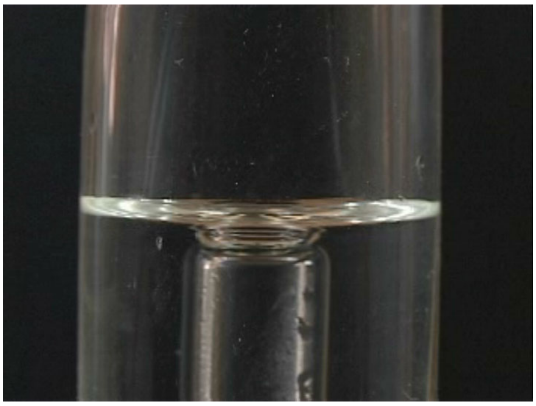


Fig. 7(e)

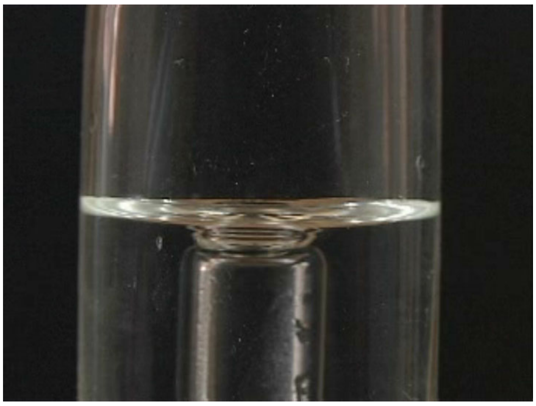


Fig. 7(f)

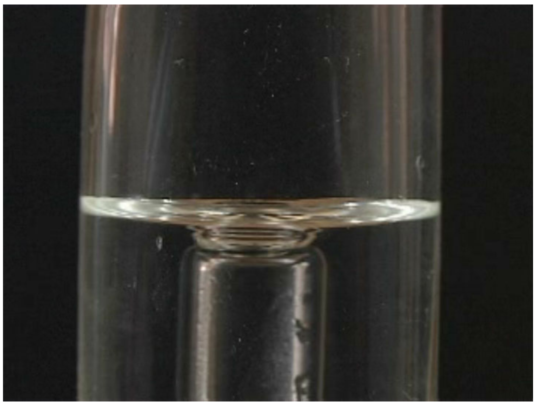


Fig. 7(g)

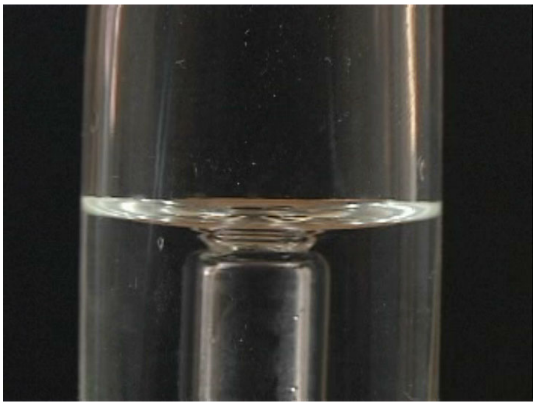


Fig. 7(h)

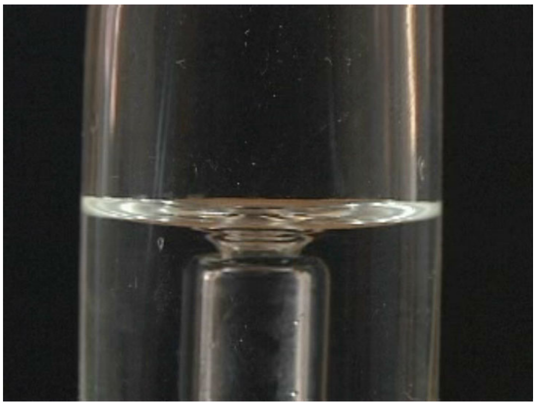


Fig. 7(i)

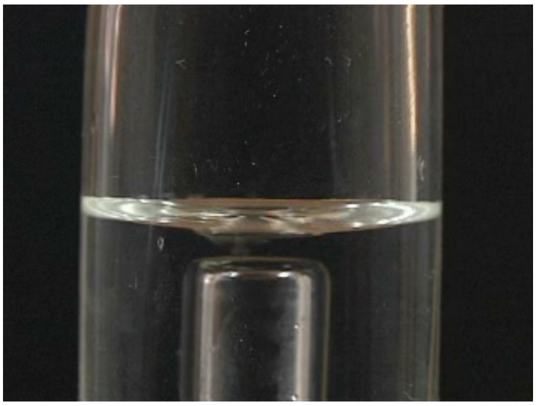


Fig. 7(j)

Fig. 7 are from a time lapse video (30 frames per second) taken with a digital camcorder. Fig. 7(a) is taken just as the diver floats to the center of the vessel. Fig. (b) ~ (g). are photographed after 2 seconds. Fig. 7(h) ~ (j) are in the time interval of 1/30 second. The fringes of capillary wave increased with depth as in Fig. (a) ~ (g).

Fig.7 (i) is 1/30 second before the water film breaks away from the diver. In Fig.7(j). the film has just broke away, as the contact angle was 0°. We can get the effective radius R of the diver from this picture, and get the maximum pressure p1 from manometer. Using equation (7) we can calculate the surface tension of water. The time interval of Fig.7 (i) to (j) is 1/30 second, the vertical displacement of the film is 0.2cm, which means that the vertical velocity of the film is 60cm/s. The phenomena of progressively shrinking rings of the diver sinking in ethyl alcohol is the same as in water, but it's surface tension is smaller than water, so, the number of fringes is less than in water. The viscosity of glycerin is 1,000 fold higher than water, the diver sinks very smoothly, so unfortunately we can't find the fringe of capillary wave.

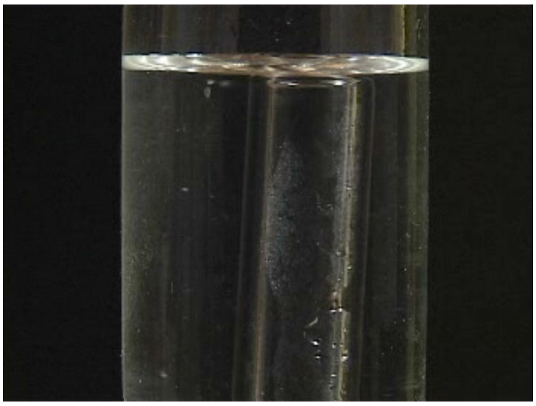


Fig. 8(a)

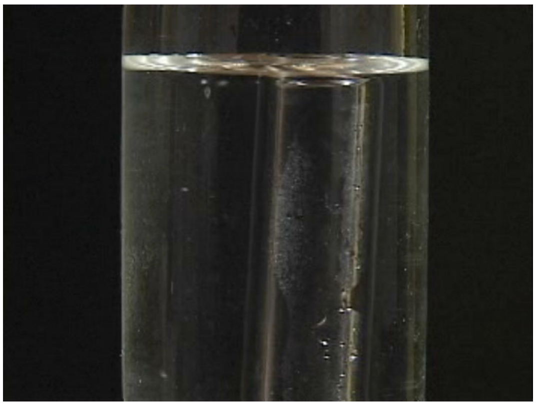


Fig. 8(b)

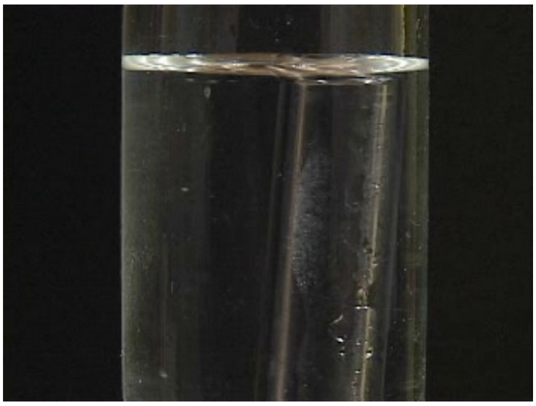


Fig. 8(c)

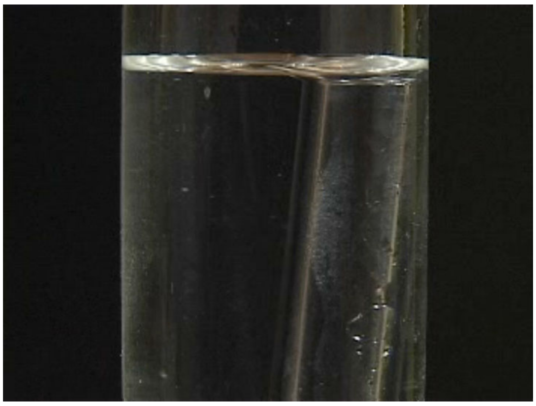


Fig. 8(d)

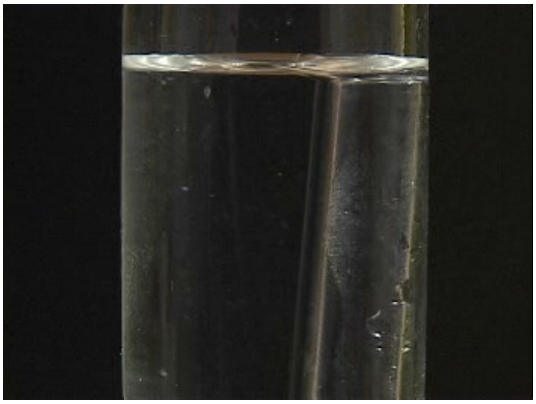


Fig. 8(e)

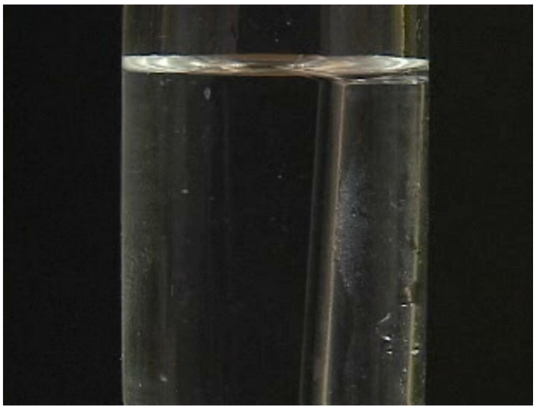


Fig. 8(f)

Fig.6 shows the Cartesian diver as it’s attracted toward the wall of the vessel. The surface on the right hand side of the diver is in the incline angle of 35° downward with horizontal. The left hand side is almost horizontal. Since the net forces acting upon the diver was to the left, the diver was attracted to the left side.

Fig.8 was taken from the video (30 frames per second). The Cartesian diver sticks to the wall of the vessel at atmospheric pressure. The top of diver is attracted toward the wall by the surface tension, where the dragging force of viscosity pulls the diver beneath the water surface. At the wall of the vessel, the meniscus effect pulls the diver toward the upper left. Fig.9 shows the displacement of the top of the diver as it floats..

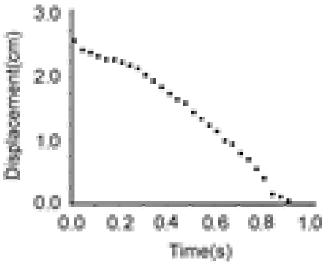


Fig.9. Displacement of the diver as a function of time

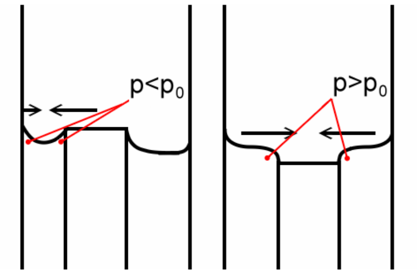
DISCUSSION

1. The measured values of surface tension have three significant figures, and within 1% that of the recognized values. The error may be due to: (1) purity of the chemical, (2) disturbance of liquid when the diver is sinking, which may be reduced by very slowly varying the pressure, and (3) contact angle of the liquid film with the diver is not controllable as it about to sink.

2. In the sinking process, the disturbance of liquid creates a capillary wave. The wave numbers increase with sinking depth of the diver. The radius of the progressive rings is reduced due to the pressure gradient. The moment at which the liquid film breaks away from the diver, the pressure reaches P_1, and the “effective radius” may be measured from Fig.7. (c) Thereby the surface tension can accurately be calculated.

3. The diver in the vessel is at atmospheric pressure P_0, as in Fig. 8(a), so that $\cot\theta_1\cot\theta_2>0$. The pressure p just inside that the meniscus of the liquid-air interface near diver and vessel is lower than atmospheric pressure P_0 ; the diver is attracted toward the wall of the vessel. When we press air to increase the pressure in the vessel, $\theta 1$ becomes larger than 90°, as shown in Fig.8. (b) The pressure near the diver is larger than P0, so that $\cot\theta_1\cot\theta_2<0$, the diver is repelled by the wall of the vessel.

4. In the attractive Cheerios effect, the diver is initially in the unbalance equilibrium and experiencing an imbalance force that triggers the motion. The upper portion of the diver is attracted toward the wall by surface tension; where as the dragging force of viscosity pulls the diver beneath the surface. At the wall of the vessel, the meniscus effect pulls the diver toward the upper right.



(a) (b)

Fig.8 Contact angles of water with the vessel and the diver are depicted in cross section diagram

CONCLUSION

The Cartesian diver is a common toy. Here, we've given it a new life. This new life enriches the Cartesian diver to show more new physics phenomena than what we previously knew, such as the Cheerios effect and capillary wave. We have also established the Cartesian diver as a surface tension indicator.

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評語

優點：利用簡單設施與原理設計出良好儀器，可測量準確之表面張力，並衍生出原理，可解釋懸浮粒子之貼壁現象與群聚之效應。

缺點：接觸角與表面張力及內聚力之關係，雖有量化分析並推論至 cheerors，但應可再作更多理論分析與實驗結果之比較。