作品名稱：From Chaos to Regularity : A study In Jet Flow 從混沌到規律

--- 噴流的研究

高中組　應用科學科　第二名

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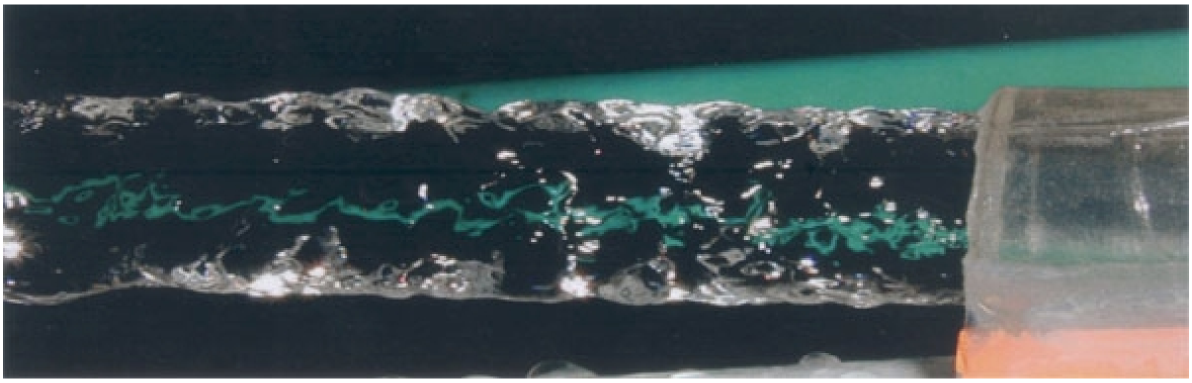
關鍵詞：REGULARITY、JET FLOW、DROP FORMATION

From Chaos to Regularity：A Study In Jet Flow



從渾沌到規律：噴流的研究

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編　號：

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附件七：說明書內文

名稱　　From Chaos to Regularity ：A Study in Jet Flow

從混沌到規律 / 對噴流的研究

內文

一、研究動機　由於對流體力學及水流至水滴形成的過程感覺好奇，於是在暑假自行研究

二、研究目的　由紊亂的水流形成小水滴是否有一定的規則 我們希望能尋找到這樣的規則，是否有一定的計算公式？噴流是否可以加以控制，而進行本項實驗

研究動機 INTRODUCTION：

What does one see when examining water flowing freely from a water faucet? Is the formation of droplets regular, containing inherent frequencies, or is it chaotic? Water is a common fluid that is essential for everyday life. The purpose of this experiment is to observe and both qualitatively and quantitatively the behavior of water, and fluids in general. This is the question that this experiment has set out to solve: whether regularity exists in a seemingly chaotic environment.

研究目的 HYPOTHESES：

1. There is a difference in the regularity of drop formation between laminar flow and turbulent flow.
2. This difference in regularity can be examined.
3. There are certain frequencies in the laminar flow that can be detected.

初步實驗 PRELIMINARY EXPERIMENTATION

In the exploratory stage of the project, we constructed a simple apparatus to observe qualitatively the behavior of a jet flow. Pictures of the water flow were taken with an Olympus C2020 digital camera (fig 2, 3). There were no apparent regularities, only unpredictable fluctuations. The goal of this initial exploratory stage was to develop a data gathering technique and to serve as an early prototype for later experiments.

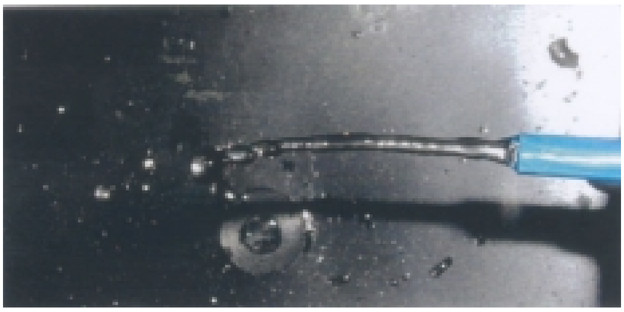


Fig.2

Fig.1

Fig.3

背景資料 A cursory review of laminar jet flow

In laminar flow, the fluid appears to move in layers, with one fluid layer sliding over the other. A turbulent flow was observed to move chaotically and with no apparent pattern. Whether flow in a pipe is laminar or turbulent is governed by the Reynolds number, which is a non-dimensional parameter.

$$\text{Reynolds number} = \frac{\text{Density} \times \text{Velocity} \times \text{Diameter}}{\text{Viscosity}}$$

When the Reynolds number is below 2000 [1], it is guaranteed that the pipe flow is laminar. When the Reynolds number is above 2000, there is no conclusion since the pipe flow would depend on other parameters as well.

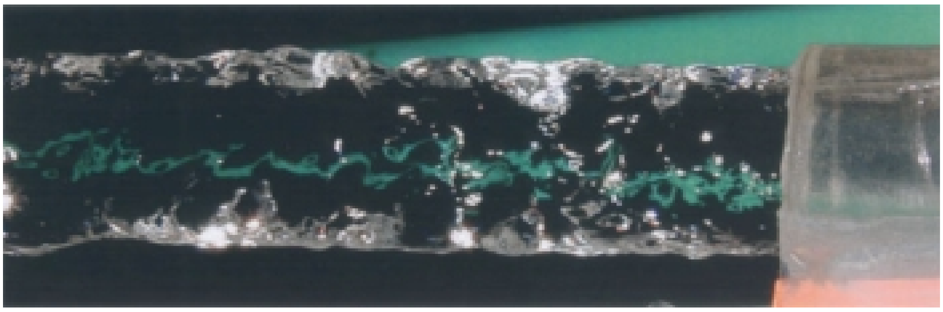


Fig.4

A vertical turbulent jet flow. It shows chaotic behavior. The diameter of the jet cross section is nearly constant. The green line in the water is the refraction of a green stand behind the jet stream.

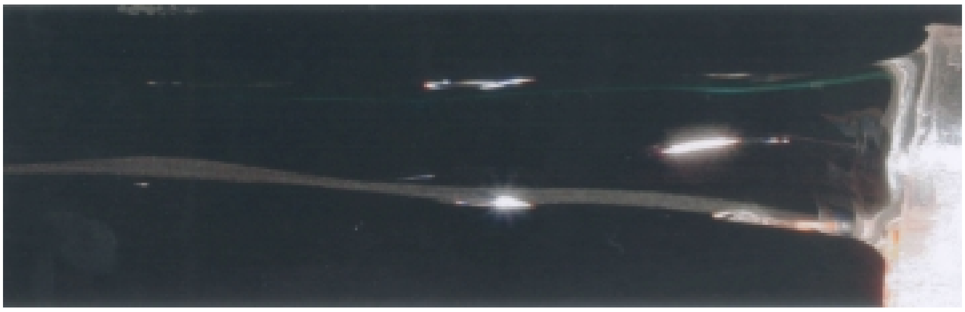


Fig.5

A vertical laminar jet flow. It shows the contraction due to the gravity and the surface tension.Notice also that the green line, the reflection of the stand behind it, much smoother than in the previous picture.

Entrance Length

When a fluid enters a tube, the initial velocity profile is uniform. Due to the viscosity and the non-slip boundary condition, or the friction of the tube walls, the fluid velocity profile will eventually be fully developed. For a fully developed laminar pipe flow, the velocity profile is parabolic, and the velocity of the center of the profile is exactly 2 times the average velocity of the flow. The length of the pipe that is required for the flow to be fully developed is the entrance length. The formula of the entrance length of a pipe flow suggested by Pao [1] is：

$$L_{ent} = 0.058 \cdot D \cdot Re \qquad (1)$$

where D is the pipe diameter and Re is the Reynolds number.

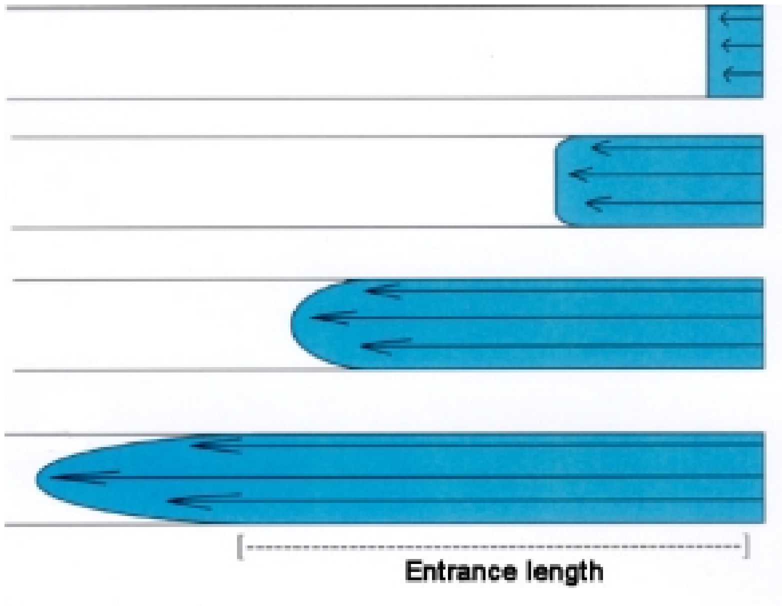


Fig.6

The Breakup Process

Before we attempt to observe the regularities in the breakup process of a jet stream, we must make a closer examination of the breakup process itself. When the fluid exits the tube, it accelerates due to gravity and the cross section of the fluid decreases. The ratio of surface area to volume increases. At a certain threshold, surface tension, along with various internal disturbances, causes the formation of water drops.

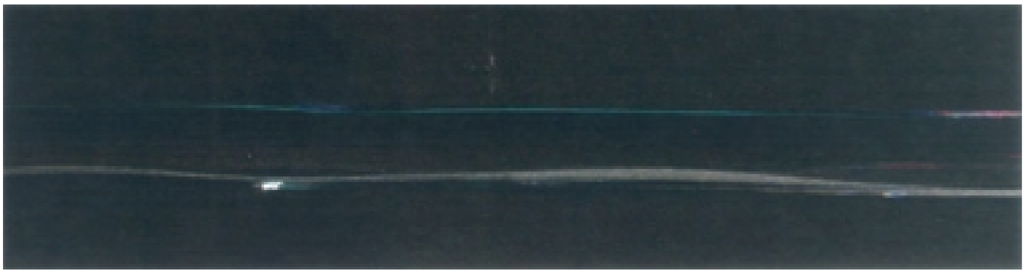


Fig.7

An example of a vertical laminar jet flow. The disturbances are still weak.

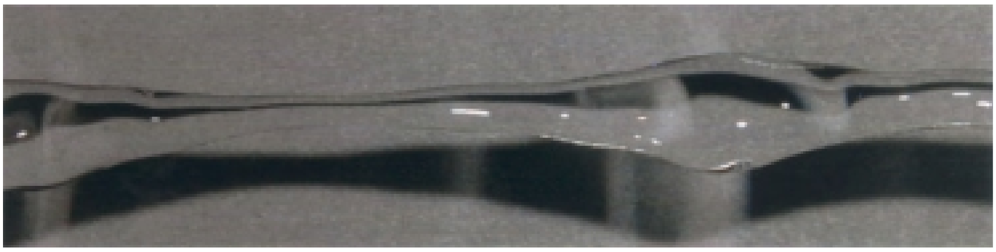


Fig.8

An example of the initial stage of drop formation.

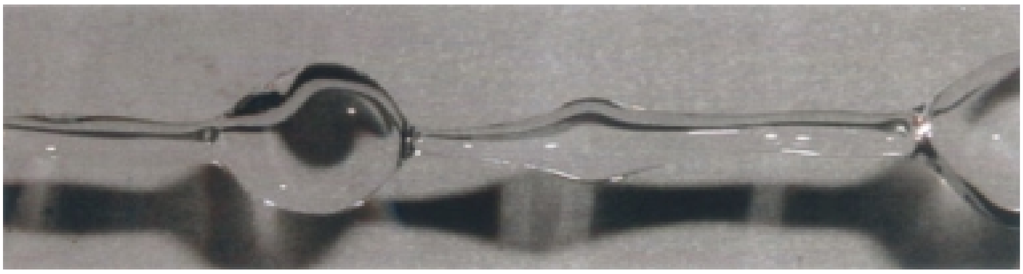


Fig.9

The continuing development of the drops. There exists interesting bulb shapes in the jet. The jet column is still connected.

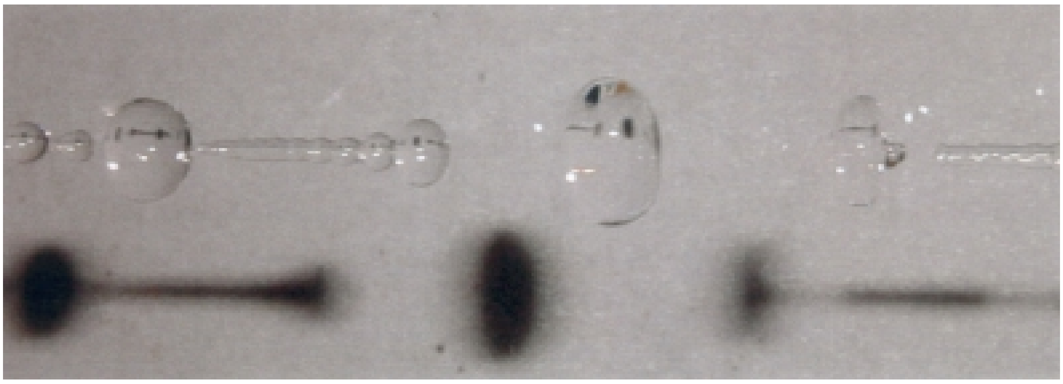


Fig. 10

The jet breaks up and turns into drops. The drops, however, do not have fixed shapes. For example, there is a big bulb in the middle that resembles a tomato, and one above it that looks like a mushroom. What follows is a string of drops that seem to be linked to another bulb. These smaller drops, called satellites, are droplets formed from the thin connecting streams between drops.

研究設備及器材 Experimental Setup.

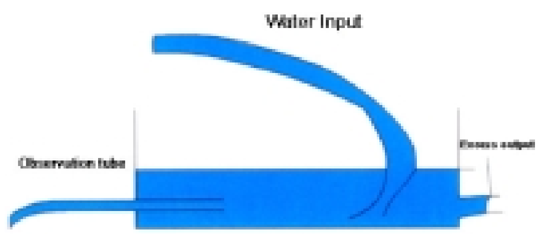
Once there was a theoretical model of the means to determine whether a flow is laminar or not, the next setup was to construct an apparatus to control the variables

Variables:
1. Pipe length
2. Pipe Diameter
3. Exit velocity
4. Density (dependent on temperature)
5. Viscosity (dependent on temperature)

Pipe length and diameter were easy to control, but since the temperature remained the same throughout the experiment, density and viscosity of the water became constant. Therefore, how to control the exit velocity became the question.

Construction of the apparatus
Main goals:
1. To control the speed of the water running through a tube.
2. To keep the water quiet of disturbances.

The best way to control the speed of the exiting water was through gravitational potential energy. By controlling the height of the water above the pipe and by using a modified version of Bernoulli's equation to account for the fluid pipe entrance frictional loss, and the energy that is required to maintain the velocity profile, we could control the exit velocity.



This design was chosen for its flexibility and control of water level, but also because of cost considerations. The entire apparatus can be but for under five hundred NT, or about seventeen US dollars. On the next page are actual photos of the apparatus.

Fig.12 & 13

研究過程與方法 Method

The apparatus operated on the principle of overflowing fluid to control the height of the water. This in turn regulated the exit velocity. Excess input water entered the system through the toweled rubber hose with slits. This made the input water quiet and kept the system free of disturbances. The control valve's height was adjusted with the fishing string on the tennis stand. Excess water left through the shorter orange hose. Thus the water level could be regulated and maintained for a long time.

This technique took some time to master, as there were several variables in establishing equilibrium between the water input and the two water outputs. Before long, the optimum flow rate was discovered. The input was discovered to be a little faster than the exiting fluid that we would be observing. The difficult part was the trial and error process, which was the process of discovering the right input flow rate and equilibrium. For example, when adjusting the height of the end of the tube, the height change of the surface level had a nearly 1 minute delay. These problems have since been dealt with and solved. The tube was made horizontal by adjusting the screw on the block of wood under the tube outside the tub. This system worked amazingly well. An input of 100cc per second was introduced, and no visible disturbances appeared on the water surface. Water level was also maintained at the desired height for any given amount of time, and this height was accurate up to l mm. A caliper was used to measure the height of the water; submerging it in the water and having one end break the surface of the water.

Despite the apparent crudity of the apparatus, it must be stressed that its design to serve two functions - to keep the water level constant and free of noise. To this end, it is very accurate and reliable. Experiments can be repeated to within 1 % accuracy. For more details on the exact materials and construction procedure, refer to Appendix A (附錄一).

討論與分析 Data Analysis and Interpretation

Breaking Point	Pipe length - 30.0 cm	Pipe length - 9.8 cm
Reynolds number ~ 2475 Exit velocity ~ 42 cm/s	Average: 11.5 cm Std. Dav. : 1.85 cm	Average: 15.8 cm Std. Dev. : 3.03 cm
Reynolds number ~ 1575 Exit velocity ~ 27 cm/s	Average: 8.61 cm Std. Dav. : 1.2 cm	Average: 9.70 cm Std. Dav. : 1.64 cm

Fig. 19

The numbers refer to the vertical distance from the exit of the pipe to the breaking point. Because the pipe was maintained completely horizontal, the vertical drop comes only from the gravitational acceleration. Consequently, the vertical distance is in direct correspondence to the time since the water has left the tube. In other words, the greater the vertical distance from pipe exit to breakup point, the longer the laminar jet flow has maintained itself. In all instances, the ambient temperature was 26 degrees Celsius, so the viscosity and density of the water stayed constant. The pipe diameter remained unchanged at 0.525 cm throughout the experiment. Therefore, the Reynolds number is a function of exit velocity, since all other variables remained constant.

At a higher Reynolds number, and therefore higher exit velocity, the breaking point was much farther from the pipe exit than at a lower Reynolds number. We believe that this was due to the greater inertial force of the faster flow, allowed the laminar jet to maintain itself for a longer period of time. Surface tension, the main force in water drop formation, could not overcome this greater inertial force as quickly, and as a result the average breakup point was 3 cm farther for the 30 cm tube, and 6.1 cm farther for the 10 cm tube.

A smaller pipe length also increased the average distance to breaking point. A possible explanation of this phenomenon lies in the properties of laminar flow. A laminar flow develops from the friction of the inner tube walls slowing down the fluid layers immediately adjacent to the tube wall. In that in a fully developed flow, the velocity at the center of the flow is 2 times of the average velocity, and the entire velocity profile is parabolic. This big difference in velocities of the inner and outer layers of the fluid causes a shearing force that induces turbulence. That turbulence quickly causes the destabilization and collapse of the flow once the fluid has exited the confines of the tube. In a less developed flow, the more uniform velocity profile maintained the jet flow for a longer period of time. It should be noted that in either case, the velocity profile was not completely parabolic, since neither tube length is sufficient for a fully developed laminar flow. However, the shorter tube had a less developed laminar flow than the longer one, and consequently, there was less shearing force and less turbulence, so the flow was maintained for a longer time.

A greater Reynolds number also means a more turbulent flow, which resulted in a greater range of breaking points as evidenced by the greater standard deviations (Fig. 16 and 19) As stated earlier, in the shorter pipe lengths, the laminar flow has had less time to develop, so the velocity profile was not parabolic. This may have also caused more variability in the breaking point, though it delayed the breakup process.

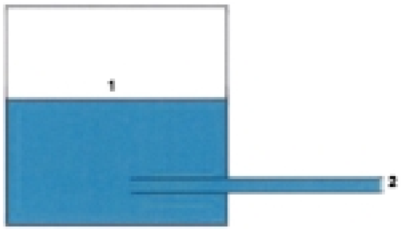
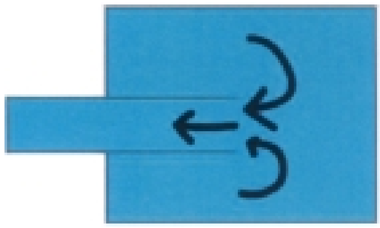
理論模式 Theoretical model

In order to find the theoretical velocity, a modified version of Bernoulli's equation was used to account for the re-entrance frictional loss the fluid encounters when the fluid enters the tube.

$$h_r + h_f = \left(\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g}\right) - \left(\frac{P_2}{\gamma} + Z_2 + \alpha\frac{V_2^2}{2g}\right) \qquad (1)$$

1 2 3 4

This is the modified version of Bernoulli's equation. Here, 3 represents the energy of the fluid at the surface of the water at (1) as seen in Fig. 14; where P is pressure, Z_1 is the gravitational potential energy at (1) Fig. 14, and Z_2 is the gravitational potential energy at (2). V is velocity, and g is gravity. Here, α represents the kinetic correction factor for laminar pipe flow, in which energy is used to sustain the velocity profile.

h_f is the frictional loss the fluid experiences when it flows through the tube and sustains its velocity profile. $h_f = \frac{64}{Re}\cdot\frac{L}{D}\cdot\frac{V_2^2}{2g}$ (2) The kinetic frictional loss inside of the tube is found by definition; D is diameter of the tube, L is length of the tube, V_2 is velocity and Re is Reynolds number	 Fig.14
h_r represents the re-entrance loss, where the fluid experiences an energy loss when it enters the tube. $h_r = C_l\frac{V_2^2}{2g}$ (3) C_l has a recommended value of 0.8 for this type of entrance loss. The Reynolds number equation is $Re = \frac{\rho\cdot V_2 D}{\mu}$ (4)	 Fig.15

Combining these 4 equations, velocity can be isolated and solved.

$$V = \frac{-64\cdot L\cdot\mu + \sqrt{4096\cdot L^2\mu^2 + 8\cdot D^4 g\cdot Z1\cdot(C1+a)\mu^2\rho^2}}{2\cdot D\cdot(C1+a)\mu^2} \qquad (5)$$

The resulting velocity can then be applied to the formula :

$$Re = \frac{\rho\cdot V_2 D}{\mu} \qquad (4)$$

This mathematical model was verified to within 4% of our experimental result, showing the consistency between actual and theoretical values. This model was later used to calculate the Reynolds numbers for consequent experiments.

研究結果 Experimental Results

To explore the factors affecting the breaking point of the jet flow, multiple pictures were taken under four different conditions. The pictures were then analyzed. These 4 photographs of figure seventeen were taken at random from the data set of Set 1. (The complete data sets are shown on page 12.) The pictures show no discernable pattern or regularity.

Pipe length = 30cm Exit velocity = 27cm/s Reynolds number = 1500

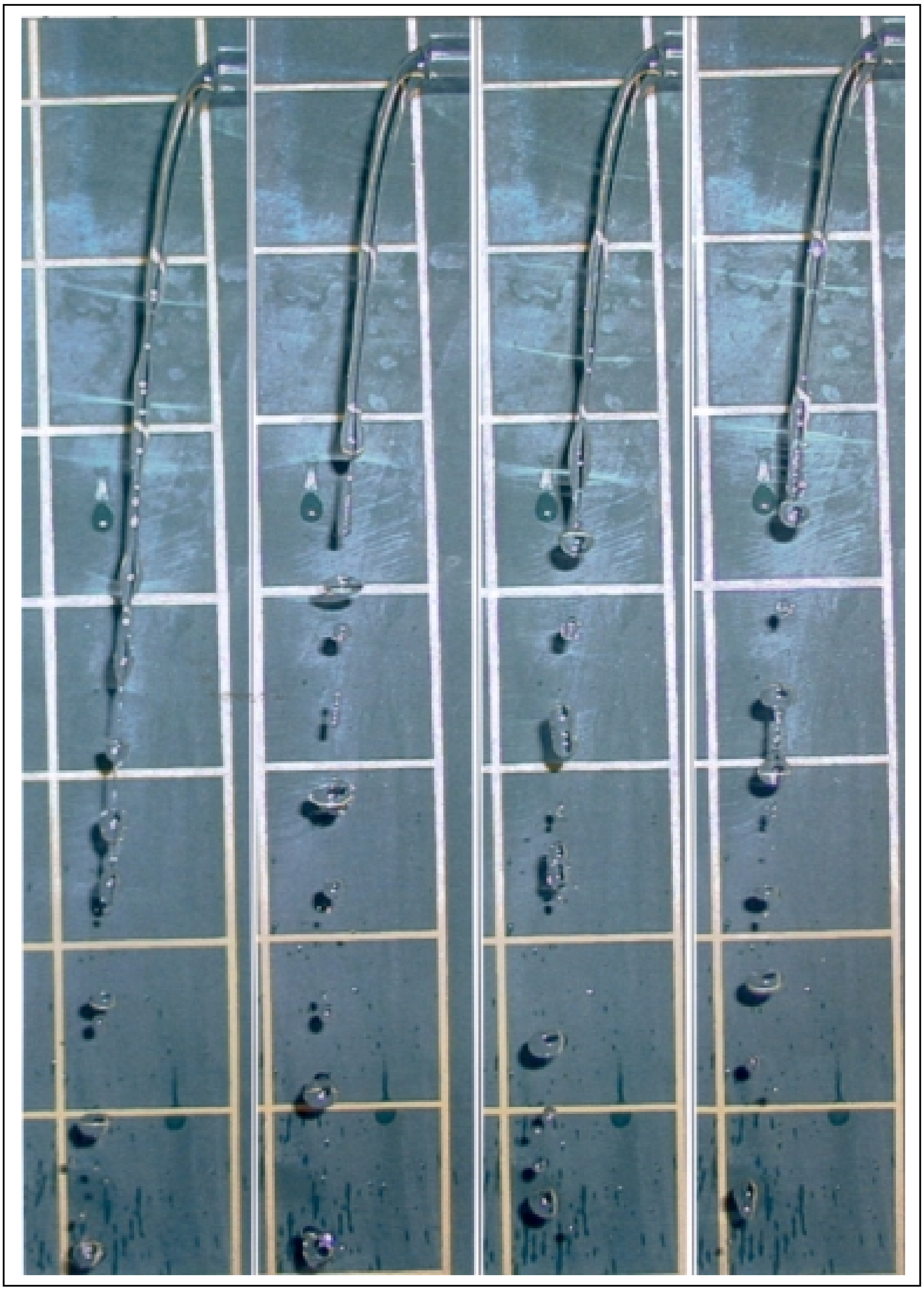


Fig.16

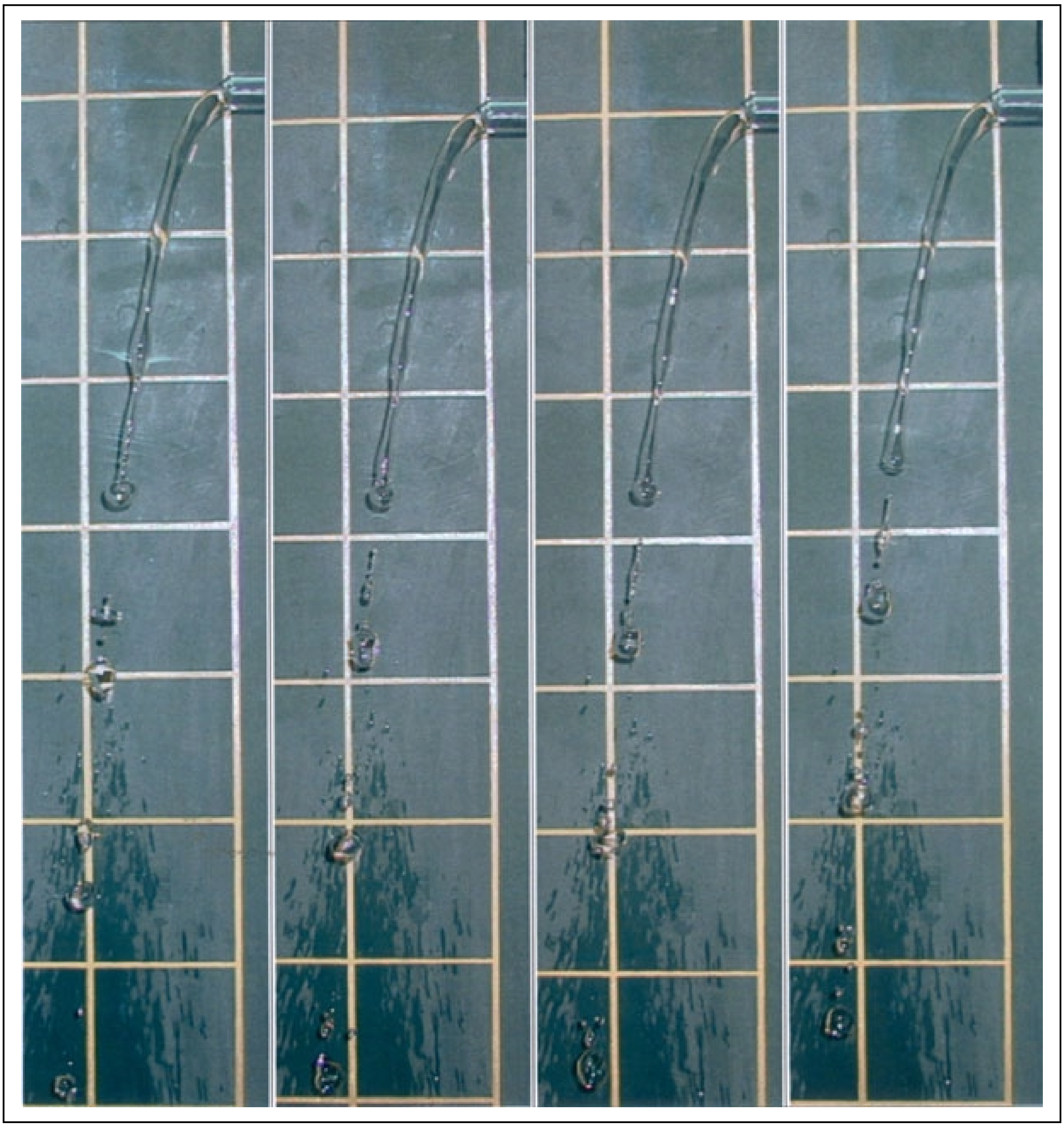


Fig. 17

These 4 photos were taken at random from the Set 3 data samples, in which pipe length equals 9.8cm, exit velocity equals 27cm/s, Reynolds number equals 1500. There is an obvious and repeating pattern in the drop formation. Note that this set has the same exit velocity and Reynolds number as the previous image set. The only difference is pipe length - the prevous set is 30 cm, and this set is 10 cm. There is a significant change in drop formation pattern. Note that not only are the drops in approximately the same relative positions, but even the size, number and position of the satellite droplets are similar.

Breaking Points

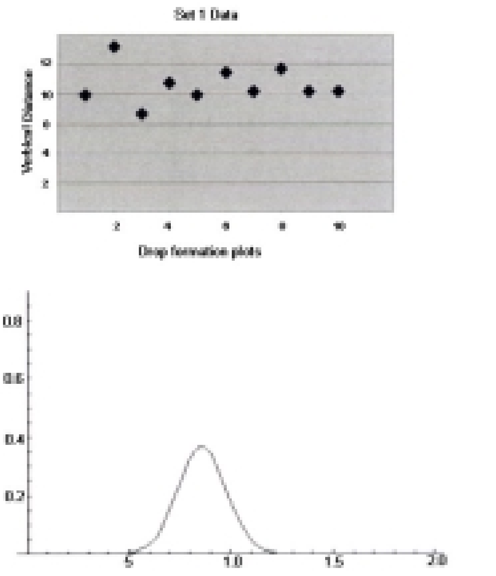
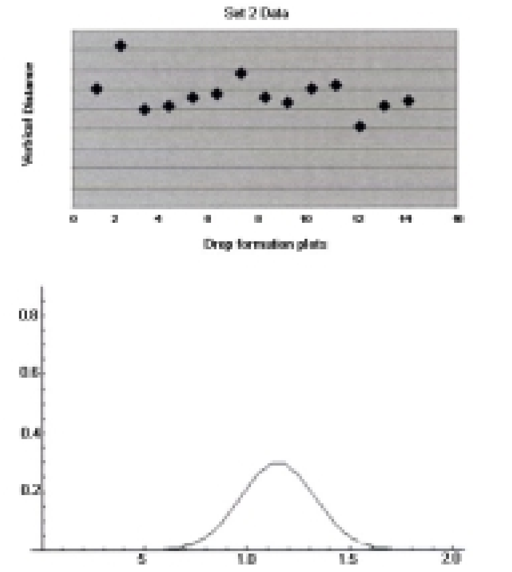
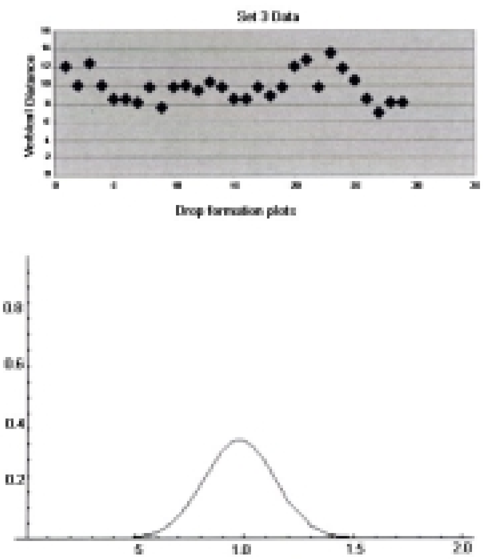
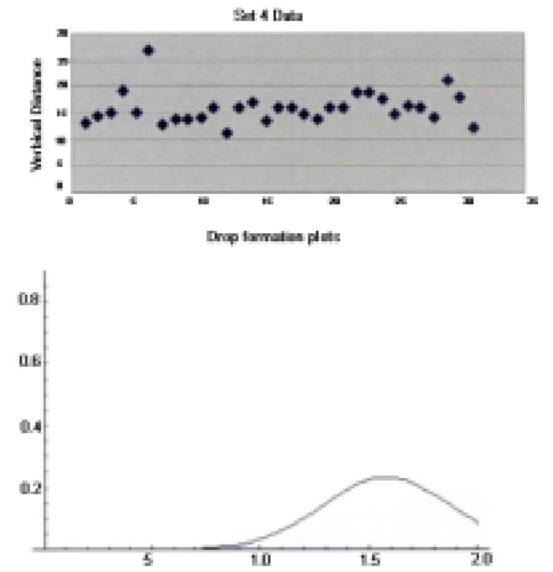
Set 1	Set 2
Set 1 Pipe length = 30cm, Exit velocity = 27cm/s Reynolds number = 1500	Set 2 Pipe length = 30cm, Exit velocity =42cm/s Reynolds Number = 2500
	
Set 3 Pipe length = 9.8cm, Exit velocity = 27cm/s Reynolds number = 1500	Set 4 Pipe length = 9.8cm, Exit velocity =42cm/s Reynolds Number = 2500
	

Fig.18

A Hidden Frequency

In observing many of the still images we captured, we realized that in almost all the situations, the drop formation was very irregular and very unpredictable, as seen in pure chaos. However, while taking pictures of a jet flow with a Reynolds number of ~ 1575, and a pipe length of 9.8 cm, there were multiple instances where the formations of drops were almost exactly identical. We also took a Quicktime video of the jet flow, and found that the water flow formation seemed to "freeze". This was because the frequency of drop formation was synchronized with the sample rate of the video, which was 15 hertz. After some simple Newtonian mechanics calculation, it was shown that the frequency of the water was 47 hertz. As seen in the pictures below, the breaking point of this flow was observed to be very regular and occurred at regular intervals. This strongly suggests an extremely patterned and regular behavior in drop formation. Strobe light synchronization with droplets is a goal worth of pursuit.

結論 CONCLUSION

The drop formation in a jet flow could be controlled in a way that regularity can emerge. It is not chaotic as originally expected. We were able to control the fluid in such a way that the flow is laminar and smooth, and thus acts in a very predictable and regular way. In our early observations, we witnessed a lot of chaos in the process of formation of droplets, we were never able to identify a pattern, or see a frequency. In our studies of theoretical fluid dynamics, we learned how to control a fluid's flow characteristics, and how the many variables directly affect the drop formation process. After building our apparatus, we controlled a flow of water so it was laminar and free of turbulence. In this we saw regularity in droplet formation, apart from the seemingly random formation in our observations of turbulent or non-laminar flows.

Future research directions include elimination of satellite droplets, which is useful in inkjet printer applications, explorations into methods to prolong laminar jet flow, and in refinement of our theoretical model, including incorporating Weber's number to estimate drop jet flow stability to determine likely breakup location by exploring surface tension and capillary action on scales where surface tension is the dominant force.

參考資料及其他 REFERENCES

..

[1] Pao, Richard H.F. (1969) Fluid Mechanics, John Wiley & Sons, Inc.

附錄一 Appendix A

Materials:

31. Standard plastic baby bathing tub	45. Tennis machine rack stand
32. 1 meter long orange rubber tubing (2cm diameter)	46. Fishing wire
33. 30cm long orange rubber tubing (2cm diameter)	47. Glass tubes
34. 1 wooden table stand	48. Nail file
35. 3 kitchen towels	49. Sandpaper
36. 1 level	50. 1 Wooden plank
37. Olympus C-2020 digital camera 2.1 mega pixel, with macro focus.	51. 1 Caliper
	52. Soldering iron
38. Slik Tripod	53. 2 rods of glue
39. Standard high school Cartesian plane blackboard	54. Pack of white polyethylene bread tie
40. 1 wooden stool	55. Wood from part of a table
41. 1 flower pot	56. Saw
42. 1 sponge	57. Screw
43. 1 glue gun	58. Screwdriver
44. 2 rulers 1 piece of wax	59. Oil based marker
	60. Paper

Construction Sequence:

13. Two holes were cut on each side of the standard bathing tub with a soldering iron. The tub was then mounted on the wooden table stand.
14. The end of the orange rubber tube was sealed with hot glue. Four to five slits were then created on the end of the tube near the sealed end.
15. This area was then wrapped with towels and attached to a faucet.
16. The hose with the towels and slits was then attached to the bottom of the tub to prevent it from floating.
17. On one side of the tub, a hole the same size as the orange rubber hosing was created. The shorter orange hose was then inserted into the hole and coagulated with hot glue.
18. Fishing string was then tied to the other end of the tube and attached onto a pulley system which was created using the tennis ball shooting machine stand.
19. On the opposite end of the tub a hole approximately 7mm wide was created with the soldering iron. This hole was fitted with tubes of various configurations that were experimented with later.
20. A glass tube was then inserted into this tube. Glass tubes of different dimensions were inserted afterwards.
21. The level was then attached to the glass tube with white polyethylene bread ties.
22. The glass tube was maintained at the right height with a block of wood and a screw that could regulate the height.
23. A high school Cartesian plane teaching board with X, Y, coordinates was placed at the end of the glass tube, where the fluid would exit.
24. An Olympus C-2020 Digital camera was mounted on a tripod and was set next to the apparatus. Macro focus was used to focus on an area of 30cm by 25cm.

評語：
本作品探討水在空氣中噴流時，外觀形態與雷諾數之間的關係。該作品含數學公式的整理與實驗道具的製作，對基本科學的暢述作很好的表達，內容也還完整。綜合之，推薦該作品受獎。