

解套—咱們的數學

高中組數學科第二名

台灣省立鳳新高級中學

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指導教師：楊天德、陳慶瑞

一、研究動機

上數學課時，老師教我們玩〈河內之塔〉、〈虧格〉，進階地玩〈解套〉又叫六子連芳。種類多勾起了好奇心，引發與師長、同學一系列的探討，看到了中國人的智慧、藝術和幽默感。

二、研究目的

解套是一種組合數學歸納遊戲，是童玩益智，困難而有趣。因種類多除了歸納以一定順序組合外，是否還有別的方法及共通性呢？再進一步探討其刻口結構變化。

三、研究器材

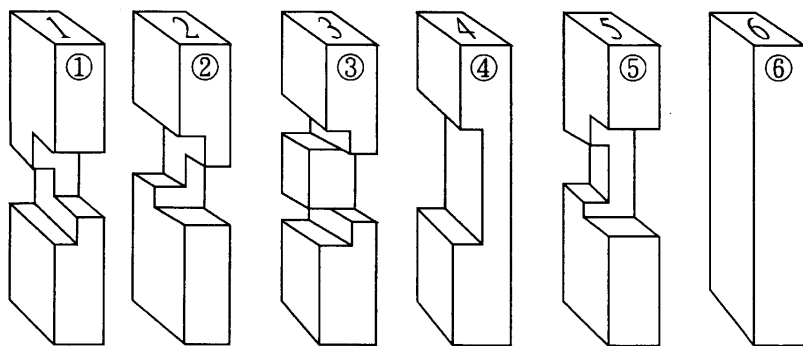
尺、筆、木塊、小鋸子、砂紙。

四、研究過程

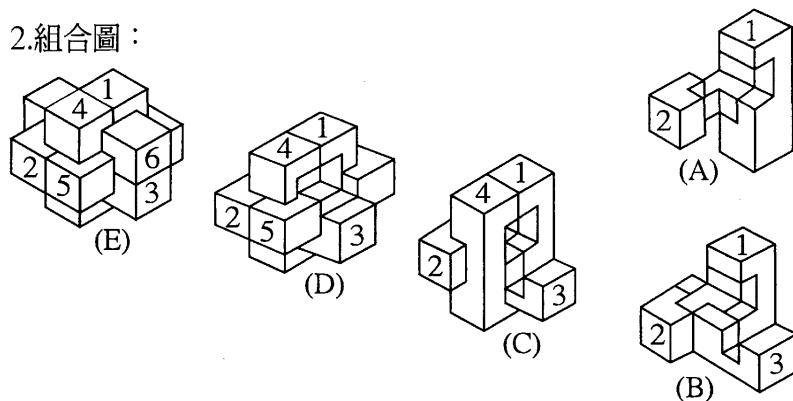
中國古代木工的榫頭銜合聞名於世。六子連芳由六塊刻口柱狀木塊組成，以一定之順序可組成立體十字之形狀。這種不用釘接的木頭組合技術是中國傳統建築的特色（尤其是廟柱的組合）曾幾何時製成玩具，而早在1836年，這種中國的組合木塊便已流行世界。現在搜集了一些圖形，有25種及解法314種，又發現了4種圖形及2種解法，共29種圖形及316種解法，在全國比賽前，又創造了木塊寅組合法，編號325，稱為「號外」，茲搜集了四種解套：名稱如下。（8分型、6分型）

甲：基本解套（8分型）

1.六塊有刻口之柱狀木塊如下：

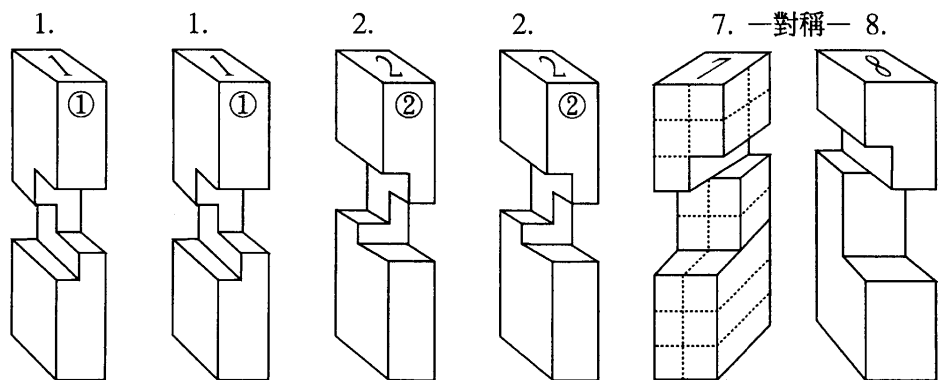


2.組合圖：

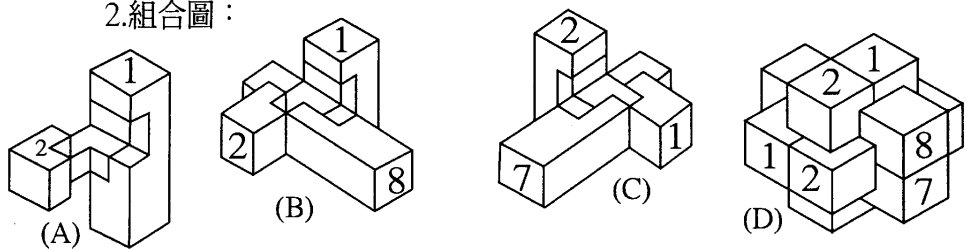


乙：三度空間對稱解套（8分型）

1.

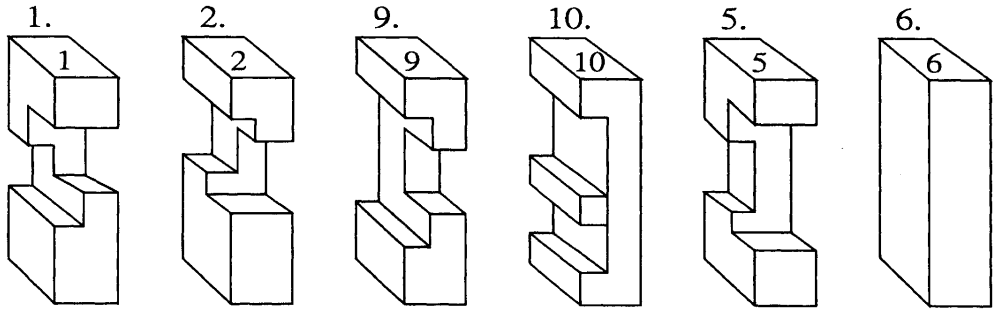


2.組合圖：

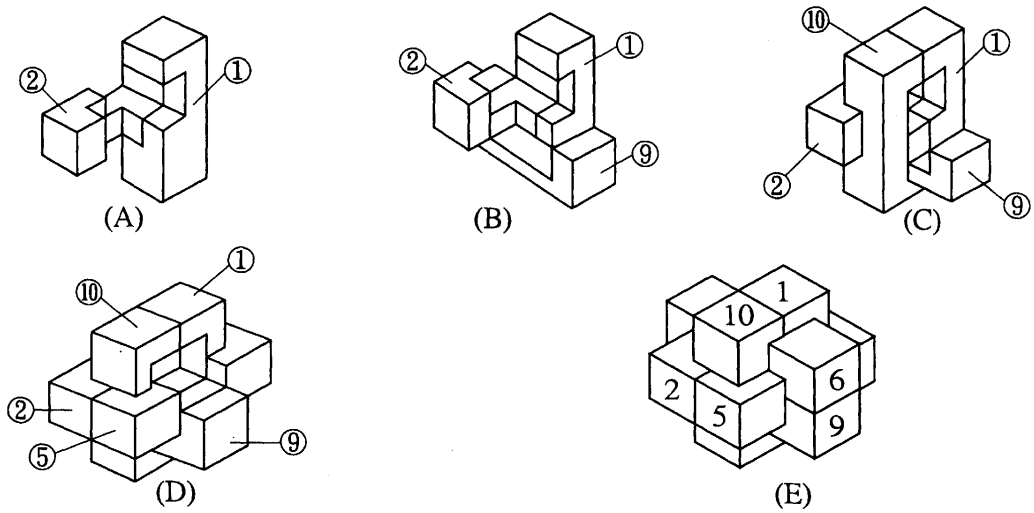


丙：改良簡易形解套（六分型）

1.

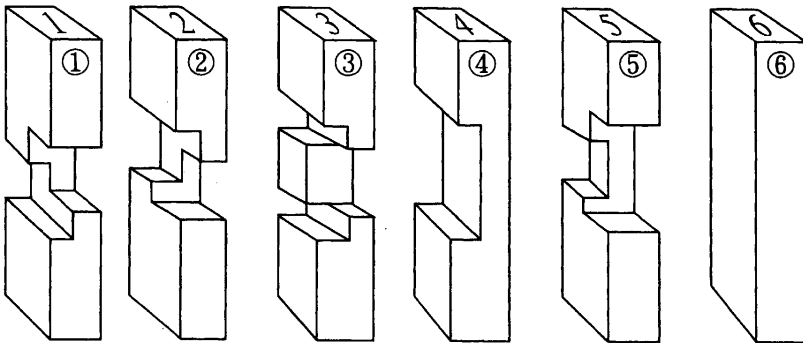


2. 組合圖：

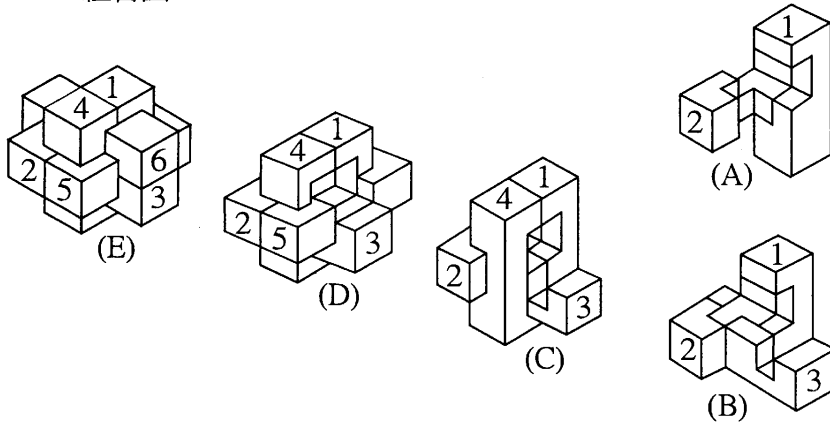


丁：中空形解套（六分型）


1.



2.組合圖：



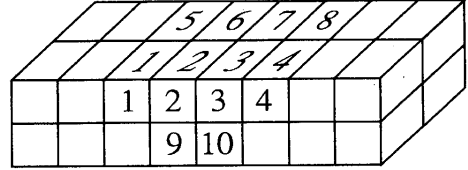
五、討論

- (一) 我們發現甲：基本——解套的每一木塊，可分為八等分，其中1號刻口木塊，如圖：如圖： 上2下3，刻口3分，如雙〈虧格〉，而2號刻口木塊如同1號左右對稱的木塊，又3號刻口木塊是上2下2，刻口4分，中2分，側看是英文字母E，正看虧格中凸出，最不容易製作，但4號刻口木塊較易製作，上2下2，中4分如古代的鎖扣。至於5號口也是上2下2，中4分但中間2半刻口，如同雙〈虧格〉對稱排列，最後6號刻口是條木狀最易製作。
- (二) 乙：三度空間對稱——解套六塊組合是用甲1-1、1-2、2塊，甲2-1、2-2、2塊，另2塊我們假設是7號、8號，它們互相對稱，其組合是第一組1-1、2-1、7號，令第二組1-2、2-2、8號。假設再裝上伸縮天線，上刻劃更佳——變為空間座標軸，利於教學。
- (三) 丙：是甲改良型——解套，將甲3號中間紅色部分取消，將它送給甲4號放於紅色虛線位置。最後丙9號與丙5號是對稱的〈也可說是一樣〉，這在製作上是方便多了，叫做改良型。改良3號設為9號，改良4號設為10號，成為五子送福了。
- (四) 丁：表面中空型——只有一塊與甲4號同，其餘皆不同，我們假設是11、12、13、14、15、16號。其中14=4較不易組合，尤其第16號最

難，要X、Y、Z軸三方向同時進行才能組合，組合後中心空六格，其組合方法是Y軸15號向右再向上，X軸12號向上再向左，Z軸14號向上，最後將16號空2刻口插入再還原，就組合成功，編號315有6解。

(五)「號外」是新發現，和丁是對稱的，組合法世界第一，因為在網路上查看不到，編號325有6解。

(六)另外將6號均分為32等分(小格)如圖(6分型的24小格)我們發現了一些規則列表如下：



甲	缺少格數	乙	缺少格數	丙	缺少格數	丁	缺少格數
1	7	1	7	1	7	11	8
2	7	1	7	2	7	12	9
3	8	2	7	9	10	13	6
4	8	2	7	10	6	14	8
5	10	7	6	5	10	15	6
6	0	8	6	6	0	16	9
總和	40	總和	40	總和	42	總和	46-6=40
解法	2	解法	3	解法	2	解法	2
編號	32, 108	編號	300,301,313	編號	152, 154	編號	315, 316

結果發現總和都是40格，所以我們就大膽假設總格數缺少40格就能組合成功。

六、結論

將四個解套，甲、乙、丙、丁分為兩組。第一組甲、乙。第二組丙、丁，第一組是將木塊8等分上2下2中4分討論、組合。第二組是將木塊6等分，上1下1中4分討論、組合。(如表一)這種不用釘接的木頭組合技術是中國傳統建築的特色。尤其是臺灣、大陸的廟宇之廟柱銜接是中國古代木工的榫頭銜合聞名於世。在英國有40個木塊組成整個出售的木塊組合遊戲。在台灣有由36塊組合的價錢約1500NT，隨便取其六塊均可組合。在其原理源自中國古代的六子連芳，依一定的順序可組合成立體十字之形狀，困難且有趣，其實更早在1836年便已流行世界

了。在古代它又叫魯班鎖，是一種空間的觀念。現在搜集了一些圖形共25種及解法共314種，又發現了4種圖形及2種解法總共29種圖形及316種解法，在全國比賽前又發現了「號外」，編號325是新發現再創新的，用排列組合對稱原理找到「寅」及新組合法有10種，總共29塊及325種組合法，如附件一及附件二，按照ABCDEF的順序組合就可成功。最後我們想用別的材料來嘗試：如〈摺紙〉、壓克力〈透明立體柱狀體〉或塑膠製品。這是下一步的構想，還在努力中，有興趣的同學不妨照圖解試試看。

表一：

甲	缺少格數	乙	缺少格數	丙	缺少格數	丁	缺少格數	號外
1	T7	1	T7	1	T7	11	Z8	Z8
2	O7	1	T7	2	O7	12	子9	寅9
3	V8	2	O7	9	Y10	13	丑6	丑6
4	J8	2	O7	10	I6	14	丁8	丁8
5	Y10	7	R6	5	Y10	15	G6	G6
6	A0	8	N6	6	A0	16	X9	W9
總和	40	總和	40	總和	40	總和	46-6=40	46-6=40
解法	2	解法	3	解法	2	解法	2	1
編號	32, 108	編號	300,301,313	編號	152, 154	編號	315, 316	325

七、參考資料

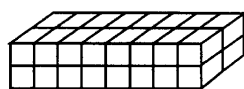
1. 高中課本第一冊。（國立編譯館）
2. 聯合報80.05.02或80.03.31第28版。
3. 廟柱。（實地觀察：左營春秋閣）
4. 魯班鎖。（聯合報86.12.25）
5. 童玩。（地球出版社）
6. 電腦網站（路）。

www.ugcs.caltech.edu/~winky/burr（附件三）

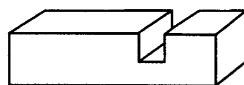
www.research.ibm.com/BurrPuzzles（附件四）

1 AI-QN-YY	61 AY-VN-GY	121 AI-WM-YX	181 DU-VF-OX	241 DN-VL-YT	301 OR-NT-OT
2 AI-QO-XY	62 AU-VM-YU	122 AI-WQ-JX	182 BG-YO-IX	242 OR-NY-DT	302 OR-NY-OT
3 AI-UR-YY	63 AX-VL-YU	123 AW-WK-QK	183 DG-VO-IX	243 DO-VN-JT	303 OT-NW-DT
4 AI-UT-YW	64 AX-VO-JU	124 AW-WP-GX	184 BU-YO-DX	244 BN-YU-YD	304 NT-YR-OD
5 AY-YQ-JY	65 AY-VF-YU	125 AG-WM-YX	185 DU-VO-DX	245 DN-VU-YD	305 OT-XR-OD
6 AH-YM-YY	66 AY-VN-JU	126 AQ-WM-QX	186 BQ-YF-WT	246 BO-YT-YD	306 NT-OW-DT
7 AE-YM-YY	67 AI-VU-YJ	127 AJ-WF-YX	187 DQ-VF-WT	247 DO-VT-YD	307 OR-XT-OD
8 AI-YF-YY	68 AW-VT-QJ	128 AJ-WN-JX	188 BM-YM-WT	248 BR-YK-OY	308 NR-OY-DT
9 AI-YN-JY	69 AJ-VR-YJ	129 AY-WF-QX	189 DM-VM-WT	249 DR-VK-OY	309 NR-YT-OD
10 AH-YU-YJ	70 AJ-VV-JJ	130 AY-WN-GX	190 BP-YL-WT	250 BR-YQ-DY	310 NT-YW-DD
11 AH-YY-JJ	71 AY-VR-QJ	131 AU-WM-YT	191 DP-VL-WT	251 DR-VQ-DY	311 OR-XY-DD
12 AI-YV-YJ	72 AX-VS-YG	132 AX-WL-YT	192 BQ-YO-IT	252 BN-YY-JD	312 NR-YY-DD
13 AI-YY-JJ	73 AY-VR-YG	133 AX-WO-JT	193 DQ-VO-IT	253 DN-VY-JD	313 NR-OT-OT
14 AX-ON-UY	74 AY-VV-JG	134 AY-WF-YT	194 BG-YT-WI	254 BO-YX-JD	314 OT-XW-DD
15 AX-SR-QY	75 AJ-TR-WY	135 AY-WN-JT	195 DG-VT-WI	255 DO-VX-JD	315 Z子丁丑GX
16 AV-WK-QY	76 AY-TR-OY	136 AJ-WT-YI	196 BU-YT-OI	256 BK-YM-OY	316 Z子丁丑GX
17 AV-WP-GY	77 AX-TS-WQ	137 AY-WT-QI	197 DU-VT-OI	257 DK-VM-OY	317 Z子丑丁GX
18 AP-WM-QY	78 AY-TR-WQ	138 AX-WU-YD	198 BG-YY-II	258 BT-YM-OY	318 Z子丑丁GX
19 AX-WF-QY	79 AJ-TT-WW	139 AT-WT-YD	199 DG-VY-II	259 DT-VF-OY	319 子丑乙丁GX
20 AX-WN-GY	80 AY-TT-WO	140 AI-YM-WX	200 BU-YY-DI	260 BT-YO-DY	320 GZ子丁丑X
21 AV-WT-QJ	81 AI-XM-WY	141 AW-YK-OX	201 DU-VY-DI	261 DT-VO-DY	321 TOVJYA
22 AX-WR-QJ	82 AW-XK-OY	142 AW-YQ-DX	202 BP-YU-WD	262 BR-YT-OJ	322 TONTOR
23 AX-WV-GJ	83 AW-XQ-DY	143 AG-YM-WX	203 DP-VU-WD	263 DR-VT-OJ	323 TORTON
24 AV-UT-OY	84 AG-XM-WY	144 AQ-YM-OX	204 BQ-YT-WD	264 BR-YY-DJ	324 TOYIYA
25 AX-UR-OY	85 AQ-XM-OY	145 AJ-YF-WX	205 DQ-VT-WD	265 DR-VY-DJ	320 Z寅丑JGW (號外)
26 AX-UT-OW	86 AJ-XF-WY	146 AY-YF-OX	206 BQ-YY-ID	266 BT-YR-OJ	
27 AV-YK-OY	87 AY-XF-OY	147 AJ-YO-IX	207 DQ-VY-ID	267 DT-VR-OJ	
28 AV-YQ-DY	88 AJ-XO-IY	148 AY-YO-DX	208 BD-MW-XY	268 BT-YM-DJ	
29 AP-YM-OY	89 AY-XO-DY	149 AU-YM-WT	209 BD-MX-YW	269 DT-VW-DJ	
30 AX-YF-OY	90 AU-XM-WU	150 AX-YL-WT	210 CD-MW-XY	270 DB-FY-YY	
31 AX-YO-DY	91 AX-XL-WU	151 AY-YF-WT	211 CD-MX-YW	271 CB-MY-YY	
32 AV-YT-OJ	92 AY-XF-WU	152 AY-YO-IT	212 DD-FW-XY	272 BB-QU-YY	
33 AX-YR-OJ	93 AY-XO-IU	153 AJ-YT-WI	213 DD-FX-YW	273 DB-RQ-YY	
34 AX-YW-DJ	94 AI-XU-WJ	154 AY-YT-OI	214 BD-UW-XJ	274 DB-RY-YJ	
35 AW-PN-YU	95 AW-XT-OJ	155 AJ-YY-II	215 DD-RW-XJ	275 DB-NY-YY	
36 AW-OP-UX	96 AJ-XR-WJ	156 AX-YU-WD	216 BN-UO-YT	276 DB-NY-JY	
37 AW-TR-YQ	97 AY-XR-OJ	157 AY-YT-WD	217 DN-RO-YT	277 BB-MY-YY	
38 AW-TT-OY	98 AJ-XW-LI	158 AY-YY-ID	218 BO-UN-YT	278 BD-MV-YY	
39 AS-XM-YU	99 AY-XW-DJ	159 BD-MY-XW	219 DO-RX-YD	279 BB-YQ-JY	
40 AV-XL-YU	100 AX-XS-WG	160 CD-MY-XW	220 BO-UX-YD	280 DB-VQ-JY	
41 AV-XO-JU	101 AY-XR-WG	161 DD-FY-XW	221 DN-RY-YD	281 CD-MV-YY	
42 AW-XF-YU	102 AY-XW-IG	162 BO-UO-XT	222 BN-UY-YD	282 DD-FV-YY	
43 AW-XN-JU	103 AV-QO-YT	163 BO-UY-XD	223 BD-UO-XY	283 DD-VN-JY	
44 AV-XS-YG	104 AW-QN-TY	164 DO-RY-XD	224 BD-QX-JW	284 BB-YU-YJ	
45 AV-XW-JG	105 AW-QO-XT	165 BT-QT-OW	225 DD-NX-JW	285 BB-YY-JJ	
46 AW-XR-YG	106 AS-YM-YT	166 BT-QY-DW	226 BR-QT-OY	286 DB-VU-YJ	
47 AW-XV-JG	107 AV-YL-YT	167 DT-NY-DW	227 DR-NT-OY	287 DB-VY-JJ	
48 AJ-NN-YY	108 AV-YO-JT	168 BE-YM-WX	228 BT-QW-DY	288 DD-VR-YJ	
49 AY-NN-UY	109 AW-YF-YT	169 DE-VM-WX	229 DT-NW-DY	289 DD-VV-JJ	
50 AJ-RR-YY	110 AW-YN-JT	170 BS-YK-OX	230 BD-QT-YW	290 DR-XK-OW	
51 AY-RR-YQ	111 AV-YU-YD	171 DS-VK-OX	231 BT-QR-OY	291 DT-XF-OW	
52 AI-VM-YY	112 AV-YY-JD	172 BS-YQ-DX	232 BR-QY-DY	292 DR-XQ-DW	
53 AI-VQ-JY	113 AW-YT-YD	173 DS-VQ-DX	233 DR-NY-DY	293 DK-XM-OW	
54 AW-VK-QY	114 AW-YX-JD	174 BM-YM-OX	234 BO-YF-YT	294 DT-XO-DW	
55 AW-VP-GY	115 AJ-ON-YX	175 DM-VM-OX	235 DO-VF-YT	295 ND-LW-XT	
56 AG-VM-YY	116 AY-ON-UX	176 BF-YM-WX	236 BN-YO-JT	296 OD-FW-XT	
57 AQ-VM-QY	117 AJ-OO-XX	177 DF-VM-WX	237 DN-VO-JT	297 ND-UW-XD	
58 AJ-VF-YY	118 AX-OO-YT	178 BG-YF-WX	238 BL-YM-YT	298 LD-MW-XT	
59 AJ-VN-JY	119 AY-ON-YT	179 DG-VF-WX	239 DL-VM-YT	299 OD-TW-XD	
60 AY-VF-QY	120 AY-OO-TX	180 BU-YF-OX	240 BN-YL-YT	300 NT-OR-OT	

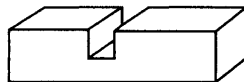
(附件一)



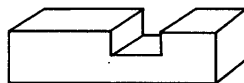
A1



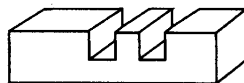
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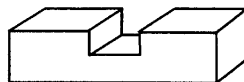
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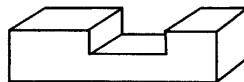
D2



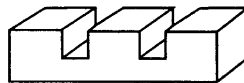
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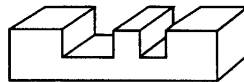
F1



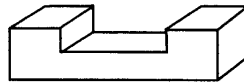
G1



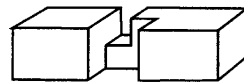
H1



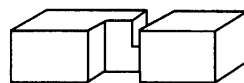
I2



J3



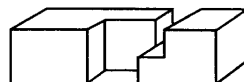
K1



L1



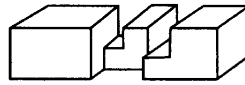
M2



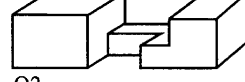
N2



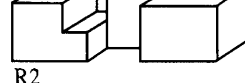
O2



P1



Q2



R2



S1



T2



U2



V2



W2



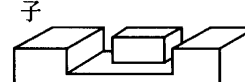
X2



Y3



Z

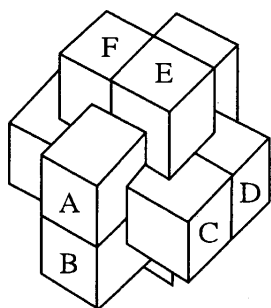


子



丑

寅



(附件二)

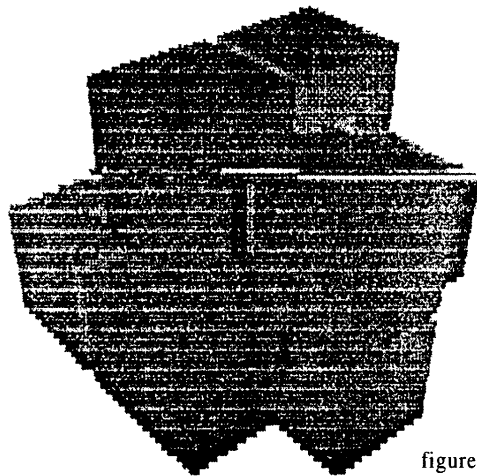


figure 1

"A burr can be roughly described as an interlocking geometrical puzzle with a high degree of external symmetry which is composed of notched rods of wood. The six-piece burr is without a doubt the best known burr. It is composed of six pieces cut from square rods and is pictured in figure 1.... There are numerous designs for the six-piece burr..." - William H. Cutler

The puzzles are most often made out of wood, but I have seen a few made out of plastic. I have made a six piece burr out of aluminum.

There exist two distinct classes of six-piece burr puzzles, one whose centers are solid, and burrs with empty spaces in them.

The solutions for burrs with solid centers were discovered by bill Cutler and written up in the *Journal of Recreational Mathematics*. In his computer search he discovered 119979 puzzle designs. Of these 314 are notchable.

"A *notchable* piece is one that can be made by making simple notches perpendicular to the axis of the rod. Notchable pieces are easily cut out with dado blades on a table saw or with a coping saw... There are no internal corners or notches parallel to the rod on notchable pieces. Mathematically speaking, each cross-section perpendicular to the axis of the rod must be convex." - William H. Cutler

There are 58 notchable pieces that are useful for the design of higher level burrs. Notchable pieces can be made with a table-saw with a dado blade.

Of the puzzles that Bill Cutler discovered, two have received considerable attention, the aesthetically pleasing, Burr #305, and the most difficult, Burr #306. Both are notchable, since all of the pieces used in them are notchable pieces.

There also exists another subset of the pieces, the *Millable* pieces. The millable set is composed of pieces which have either notchable cuts, or in which vacant spaces have vacant spaces to either side in the direction of the axis of the rod. These are the pieces that can be made with a router or a mill.

I Gentle Introduction To Burr Puzzles

have found 19 millable pieces that are not notchable.

"The recent activity in designing ever more entertaining (meaning, to some, fiendishly difficult) burrs has shifted attention to burrs that do not come directly apart (or go directly together), but rather involve the shifting back and forth of pieces, or groups of pieces, within the partially assembled burr. Some of these are so baffling as to discourage a professional locksmith... They all necessarily have one or more internal voids." - Stewart T. Coffin

These baffling puzzles are called higher level burrs.

The level of a burr is the number of moves that it takes to remove the first piece. It is common to also say how many moves it takes to remove the second piece if it cannot be removed immediately. My level 5.4 burr then takes five moves to remove the first piece, then four moves to remove the second piece once the first piece is remove.

All possible burr puzzle designs were tabulated by Bill Cutler several years ago. While statistical information from his search provides statistical information on burr puzzles in general, the number of possible designs were too large to save all designs. It seems that nobody has yet examined the millable puzzles. things called my own in these pages were developed independently of Bill Cutler's research, but several years after Bill Cutler.

(附件四)

<http://www.research.ibm.com/BurrPuzzles>

The burr puzzles site

Background
information about
burrs

Explore
precalculated burrs

Calculat
your own burr

A Burr Puzzle consists of at least three rods intersecting each other at right angles. The most famous and well known type is the six piece burr where three sets of two rods intersect each other.

At this site we let you explore six piece burrs. You can select either from a list of a few precalculated burrs, or you can select six pieces of your own making and then have us

Bibliography & Links

calculate the possible solutions for you.

Note, that you will need a Java enabled browser to visit some pages of this site. The image after a hyper-link indicates which pages require a Java enabled browser. Virtually all pages require JavaScript to properly display.

Feedback

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評語

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