

# $n$ 位數的奇妙

國中組數學科第二名

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## 一、研究動機

任意寫出一個三位數，求此三位數減去其倒寫所成三位數之差，再加上此差倒寫所成的三位數，其結果都是定值 1089，是不是任意的  $n$  位數，照這樣演算都會有一定值存在呢？

如：	4541	7302	3441	6113	8235	2031
	- 1454	- 2037	- 1443	- 3116	- 5328	- 1302
	3087	5265	1998	2997	2907	0729
	+ 7803	+ 5625	+ 8991	+ 7992	+ 7092	+ 9270
	10890	10890	10989	10989	9999	9999

似乎均會等定值，這引起了我們莫大的研究興趣。

## 二、研究過程

### (一) 符號的意義

1. 設  $a_k \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ,  $k = 1, 2, 3 \dots n$ 。

$n$  位數  $a_n \times 10^{n-1} + a_{n-1} \times 10^{n-2} + \dots + a_2 \times 10 + a_1$  以符號  $[a_n, a_{n-1}, \dots, a_2, a_1]$  表之

其倒寫  $a_1 \times 10^{n-1} + a_2 \times 10^{n-2} + \dots + a_{n-1} \times 10 + a_n$  以符號  $[a_1, a_2, \dots, a_{n-1}, a_n]$  表之

2. 在  $[a_n, a_{n-1}, \dots, a_2, a_1] - [a_1, a_2, \dots, a_{n-1}, a_n]$  中，令  $a_n > a_1$ ，且其差以  $[b_n, b_{n-1}, \dots, b_2, b_1]$  表之

3. 形如 003, 0122, 00014, …… 均視為三位數，四位數，五位數

，且其倒寫爲 300,2210,41000 ·····

4. 令  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

(一) 二位數  $a_2 \times 10 + a_1$

$$[b_2 b_1] = a_2 \times 10 + a_1 - (a_1 \times 10 + a_2) = (10-1)(a_2 - a_1)$$

令  $a_2 - a_1 = k$ ,  $k \in A$

$$\text{則 } [b_2 b_1] = (10-1)k \Rightarrow [b_1 b_2] = 9(11-k)$$

$$\therefore [b_2 b_1] + [b_1 b_2] = (10-1)k + 9(11-k) = 9 \times 11 = 99 = 11(10-1)$$

(二) 三位數  $a_3 \times 10^2 + a_2 \times 10 + a_1$

$$\begin{aligned} [b_3 b_2 b_1] &= (a_3 \times 10^2 + a_2 \times 10 + a_1) - (a_1 \times 10^2 + a_2 \times 10 + a_3) \\ &= (10^2 - 1)(a_3 - a_1) \end{aligned}$$

令  $a_3 - a_1 = k$ , 則  $[b_3 b_2 b_1] = (10^2 - 1)k$ ,  $k \in A$

$$\begin{aligned} k = 1 \text{ 時}, \quad [b_3 b_2 b_1] &= 10^2 - 1 = 99 \Rightarrow [b_1 b_2 b_3] = 990 \\ &= 10(10^2 - 1) \end{aligned}$$

$$\begin{aligned} k = 2 \text{ 時}, \quad [b_3 b_2 b_1] &= 2(10^2 - 1) = 198 \Rightarrow [b_1 b_2 b_3] \\ &= 891 = 9(10^2 - 1) \end{aligned}$$

$$\begin{aligned} k = 3 \text{ 時}, \quad [b_3 b_2 b_1] &= 3(10^2 - 1) = 297 \Rightarrow [b_1 b_2 b_3] \\ &= 792 = 8(10^2 - 1) \end{aligned}$$

⋮ ⋮

$$\begin{aligned} k = 9 \text{ 時}, \quad [b_3 b_2 b_1] &= 9(10^2 - 1) = 891 \Rightarrow [b_1 b_2 b_3] \\ &= 198 = 2(10^2 - 1) \end{aligned}$$

$$\therefore [b_3 b_2 b_1] = k(10^2 - 1) \Rightarrow [b_1 b_2 b_3] = (11-k)(10^2 - 1)$$

$$\begin{aligned} \therefore [b_3 b_2 b_1] + [b_1 b_2 b_3] &= k(10^2 - 1) + (11-k)(10^2 - 1) \\ &= 11(10^2 - 1) = 1089 \end{aligned}$$

(四) 四位數  $a_4 \times 10^3 + a_3 \times 10^2 + a_2 \times 10 + a_1$

$$\begin{aligned} [b_4 \cdots b_1] &= a_4 \times 10^3 + a_3 \times 10^2 + a_2 \times 10 + a_1 - (a_1 \times \\ &\quad 10^3 + a_2 \times 10^2 + a_3 \times 10 + a_4) = (10^3 - 1)(a_4 - a_1) \\ &\quad + (10^2 - 10)(a_3 - a_2) \end{aligned}$$

設  $a_4 - a_1 = x$ ,  $|a_3 - a_2| = y$ ,  $x \in A$ ,  $y \in A$

1. 若  $a_3 > a_2$ , 則  $[b_4 \cdots b_1] = (10^3 - 1)x + (10^2 - 10)y$

$$x = 1 \text{ 時}, y = 1 \quad [b_4 \cdots b_1] = (10^3 - 1) + (10^2 - 10) = 1089$$

$$\begin{aligned}
& \Rightarrow [b_1 \cdots b_4] = 9801 = 10^4 - 2 \times 10^2 + 0 \times 10 + 1 \\
y=2 \quad & [b_4 \cdots b_1] = (10^3 - 1) + 2(10^2 - 10) = 1179 \\
& \Rightarrow [b_1 \cdots b_4] = 9711 = 10^4 - 3 \times 10^2 + 10 + 1 \\
y=3 \quad & [b_4 \cdots b_1] = (10^3 - 1) + 3(10^2 - 10) = 1269 \\
& \Rightarrow [b_1 \cdots b_4] = 9621 = 10^4 - 4 \times 10^2 + 2 \times 10 + 1 \\
& \vdots \qquad \vdots \\
y=9 \quad & [b_4 \cdots b_1] = (10^3 - 1) + 9(10^2 - 10) = 1809 \\
& \Rightarrow [b_1 \cdots b_4] = 9081 = 10^4 - 10 \times 10^2 + 8 \times 10 + 1 \\
\therefore x=1, y \in A, \quad & [b_1 \cdots b_4] = 10^4 - (y+1) \times 10^2 + (y-1) \times 10 + 1 \\
x=2 \text{ 時}, y=1 \quad & [b_4 \cdots b_1] = 2(10^3 - 1) + (10^2 - 10) = 2088 \\
& \Rightarrow [b_1 \cdots b_4] = 8802 = 9 \times 10^3 - 2 \times 10^2 + 0 \times 10 + 2 \\
y=2 \quad & [b_4 \cdots b_1] = 2(10^3 - 1) + 2(10^2 - 10) = 2178 \\
& \Rightarrow [b_1 \cdots b_4] = 8712 = 9 \times 10^3 - 3 \times 10^2 + 1 \times 10 + 2 \\
& \vdots \qquad \vdots \\
y=9 \quad & [b_4 \cdots b_1] = 2(10^3 - 1) + 9(10^2 - 10) = 2808 \\
& \Rightarrow [b_1 \cdots b_4] = 8082 = 9 \times 10^3 - 10 \times 10^2 + 8 \times 10 + 2 \\
\therefore x=2, y \in A \text{ 時}, \quad & [b_1 \cdots b_4] = (11-2) \times 10^3 - (y+1) \times 10^2 \\
& \quad + (y-1) \times 10 + 2
\end{aligned}$$

仿上可推知  $x \in A, y \in A$

$$\begin{aligned}
& [b_1 \cdots b_4] = (11-x) \times 10^3 - (y+1) \times 10^2 + (y-1) \times 10 + x \\
\therefore [b_4 \cdots b_1] + [b_1 \cdots b_4] & = (10^3 - 1)x + (10^2 - 10)y + (11 - x) \times 10^3 - (y+1) \times 10^2 + (y-1) \times 10 + x \\
& = 11 \times 10^3 - 10^2 - 10 = 11(10^2 - 1) \times 10 = 10890
\end{aligned}$$

2. 若  $a_3 = a_2$ , 則  $[b_4 \cdots b_1] = (10^3 - 1)x$

$$\begin{aligned}
& \Rightarrow [b_1 \cdots b_4] = 999(11-x) = 10^4 - (x-1) \times 10^3 - 10 + (x-1) \\
\therefore [b_4 \cdots b_1] + [b_1 \cdots b_4] & = (10^3 - 1)x + 10^4 - (x-1) \times 10^3 - 10 + (x-1) = 10^4 + 10^3 - 10 - 1 = 11(10^2 - 1)
\end{aligned}$$

$$(10^3 - 1) = 10989$$

3. 若  $a_3 < a_2$ , 則  $[b_4 \dots b_1] = (10^3 - 1)x - (10^2 - 10)y$ ,  $x \in A$ ,  $y \in A$  仿上可推知  $[b_1 \dots b_4] = 10^4 - x \times 10^3 + y \times 10^2 - y \times 10 + (x - 1)$

$$\begin{aligned} \therefore [b_4 \dots b_1] + [b_1 \dots b_4] &= (10^3 - 1)x - (10^2 - 10)y + 10^4 \\ &\quad - x \times 10^3 + y \times 10^2 - y \times 10 + x - 1 \\ &= 10^4 - 1 = 9999 \end{aligned}$$

由以上討論知四位數有 10890, 10989, 與 9999 三種定值

(五) 五位數  $a_5 \times 10^4 + a_4 \times 10^3 + a_3 \times 10^2 + a_2 \times 10 + a_1$

$$[b_5 \dots b_1] = (10^4 - 1)(a_5 - a_1) + (10^3 - 10)(a_4 - a_2)$$

設  $a_5 - a_1 = x$ ,  $|a_4 - a_2| = y$ ,  $x \in A$ ,  $y \in A$

$$1. a_4 > a_2, \text{ 則 } [b_5 \dots b_1] = (10^4 - 1)x + (10^3 - 10)y$$

仿四位數之 1, 可推知

$$\begin{aligned} [b_1 \dots b_5] &= (11 - x) \times 10^4 - (y + 1) \times 10^3 + 9 \times 10^2 \\ &\quad + (y - 1) \times 10 + x \end{aligned}$$

$$\begin{aligned} \therefore [b_5 \dots b_1] + [b_1 \dots b_5] &= 11 \times 10^4 - 10^3 + 9 \times 10^2 - 10 \\ &= 11 \times 10^4 - 10^2 - 10 = 11 \times (10^3 - 1) \times 10 \\ &= 11 \times (10^4 - 10) = 109890 \end{aligned}$$

2. 若  $a_4 = a_2$ , 則  $[b_5 \dots b_1] = (10^4 - 1)x$ ,  $x \in A$

$$\Rightarrow [b_1 \dots b_5] = (11 - x) \times 10^4 - (11 - x)$$

$$\begin{aligned} \therefore [b_5 \dots b_1] + [b_1 \dots b_5] &= (10^4 - 1)x + (11 - x) \times 10^4 \\ &\quad - (11 - x) = 11 \times 10^4 - 11 = 11 \times (10^4 - 1) \\ &= 109989 \end{aligned}$$

3. 若  $a_4 < a_2$ , 則  $[b_5 \dots b_1] = (10^4 - 1)x - (10^3 - 10)y$ ,

$$\begin{aligned} x \in A, y \in A \Rightarrow [b_1 \dots b_5] &= 10^5 - x \times 10^4 + (y - 1) \times 10^3 \\ &\quad + 10^2 - y \times 10 + (x - 1) \end{aligned}$$

$$\begin{aligned} \therefore [b_5 \dots b_1] + [b_1 \dots b_5] &= 10^5 - 10^3 + 10^2 - 1 = (10^2 - 1) \\ &\quad (10^3 + 1) = 99099 \end{aligned}$$

由以上討論知, 五位數有 109890, 109989, 99099 三種定值。

(六)  $n$  位數之探討 (在  $a_n > a_1$  之情形下)

$$[b_n \cdots b_1] = (10^{n-1} - 1)(a_n - a_1) + (10^{n-2} - 10)(a_{n-1} - a_2) \\ + \cdots + (10^{n-k} - 10^{k-1})(a_{n-k+1} - a_k)$$

$$k=1, 2, 3 \cdots m \quad (n=2m \text{ 或 } 2m+1)$$

$$\text{設 } |a_{n-k+1} - a_k| = x_k, k=1, 2, 3 \cdots n, x_k \in A$$

今就  $a_{n-k+1}$  與  $a_k$  之大小中 1.  $a_{n-k+1} > a_k$ , 2.  $a_{n-k+1} = a_k$ , 3.

$a_{n-k+1} < a_k$  三特殊情形討論如下

1.  $a_{n-k+1} > a_k$

$$\text{此時 } [b_n \cdots b_1] = (10^{n-1} - 1)x_1 + (10^{n-2} - 10)x_2 + \cdots + \\ (10^{n-k} - 10^{k-1})x_k$$

$$(1) n=6, [b_6 \cdots b_1] = (10^5 - 1)x_1 + (10^4 - 10)x_2 + (10^3 - \\ - 10^2)x_3$$

$$\Rightarrow [b_1 \cdots b_6] = (11 - x_1)10^5 - (x_2 + 1) \times 10^4 + (9 - x_3) \\ \times 10^3 + (x_3 - 1) \times 10^2 + x_2 \times 10 + x_1$$

$$\therefore [b_6 \cdots b_1] + [b_1 \cdots b_6] = 11 \times 10^5 - 10^4 + 9 \times 10^3 - 10^2 = 10^2 \\ (11 \times 10^3 - 10^2 + 9 \times 10 - 1) = 10^2(11 \times 10^3 \\ - 1) = 11 \times (10^3 - 1) \times 10^2 = 1098900 \\ = 11 \times (10^5 - 10^2)$$

$$(2) n=7, [b_7 \cdots b_1] = (10^6 - 1)x_1 + (10^5 - 10)x_2 + (10^4 - 10^2) \\ x_3$$

$$\Rightarrow [b_1 \cdots b_7] = (11 - x_1) \times 10^6 - (x_2 + 1) \times 10^5 + (9 - x_3) \\ \times 10^4 + 9 \times 10^3 + (x_3 - 1) \times 10^2 + x_2 \times 10 \\ + x_1$$

$$\therefore [b_7 \cdots b_1] + [b_1 \cdots b_7] = 11 \times 10^6 - 10^5 + 9 \times 10^4 + 9 \times 10^3 - 10^2 \\ = 11(10^4 - 1) \times 10^2 = 10998900 \\ = 11 \times (10^6 - 10^2)$$

$$(3) n=8, [b_8 \cdots b_1] = (10^7 - 1)x_1 + (10^6 - 10)x_2 + (10^5 - \\ 10^2)x_3 + (10^4 - 10^3)x_4$$

$$\Rightarrow [b_1 \cdots b_8] = (11 - x_1)10^7 - (x_2 + 1)10^6 + (9 - x_3)10^5 \\ (9 - x_4)10^4 + (x_4 - 1)10^3 + x_3 \times 10^2 +$$

$$x_2 \times 10 + x_1$$

$$\begin{aligned}\therefore [b_8 \cdots b_1] + [b_1 \cdots b_8] &= 11 \times 10^7 - 10^6 + 9 \times 10^5 + 9 \times 10^4 - 10^3 \\&= 10^3(11 \times 10^4 - 11) = 11(10^4 - 1) \times 10^3 \\&= 11 \times (10^7 - 10^3)\end{aligned}$$

$$(4) n=9, [b_9 \cdots b_1] = (10^8 - 1)x_1 + (10^7 - 10)x_2 + (10^6 - 10^2)$$

$$x_3 + (10^5 - 10^3)x_4$$

$$\begin{aligned}\Rightarrow [b_1 \cdots b_9] &= (11 - x_1)10^8 - (x_2 + 1)10^7 + (9 - x_3)10^6 \\&\quad + (9 - x_4)10^5 + 9 \times 10^4 + (x_4 - 1)10^3 + \\&\quad x_3 \times 10^2 + x_2 \times 10 + x_1\end{aligned}$$

$$\begin{aligned}\therefore [b_9 \cdots b_1] + [b_1 \cdots b_9] &= 11 \times 10^8 - 10^7 + 9 \times 10^6 + 9 \times 10^5 + \\&\quad 9 \times 10^4 - 10^3 = 11 \times 10^8 - 10^4 - 10^3 \\&= 11(10^8 - 10^3) = 1099989000\end{aligned}$$

(5) 一般情形，分  $n$  為偶數，奇數討論

若  $n$  為偶數，令  $n = 2m, m \in \mathbb{N}$

$$\begin{aligned}[bn \cdots b_1] &= (10^{2m-1} - 1)x_1 + (10^{2m-2} - 10)x_2 + \dots \\&\quad + (10^m - 10^{m-1})x_m, x_k \in A \\&\Rightarrow [b_1 \cdots b_n] = (11 - x_1)10^{2m-1} - (x_2 + 1)10^{2m-2} + (9 - x_3) \\&\quad 10^{2m-3} + (9 - x_4)10^{2m-4} + \dots + (9 - x_{m-1}) \\&\quad 10^{m+1} + (9 - x_m)10^m + (x_m - 1)10^{m-1} + x_{m-1} \times \\&\quad 10^{m-2} + x^{m-2} \times 10^{m-3} + \dots + x_3 \times 10^2 + x_2 \times 10 \\&\quad + x_1\end{aligned}$$

$$\begin{aligned}\therefore [b_n \cdots b_1] + [b_1 \cdots b_n] &= 11 \times 10^{2m-1} - 10^{2m-2} + 9 \times 10^{2m-3} + \\&\quad 9 \times 10^{2m-4} + \dots + 9 \times 10^{m+1} + 9 \times 10^m - 10^{m-1} \\&= 11 \times 10^{2m-1} - 10^m - 10^{m-1} = 11 \times (10^{2m-1} - 10^{m-1}) \\&= 11(10^{n-1} - 10^{\frac{n-2}{2}})\end{aligned}$$

若  $n$  為奇數，令  $n = 2m+1, m \in \mathbb{N}$

$$[b_n \cdots b_1] = (10^{2m} - 1)x_1 + (10^{2m-1} - 10)x_2 + \dots + (10^{m+1} -$$

$$10^{m-1})x_m, x_k \in A$$

$$\begin{aligned}\Rightarrow [b_1 \cdots b_n] &= (11 - x_1)10^{2m} - (x_2 + 1)10^{2m-1} + (9 - x_3)10^{2m-2} \\&\quad + (9 - x_4)10^{2m-3} + \dots + (9 - x_{m-1})10^{m+2} + (9 -$$

$$\begin{aligned}
& x_m)10^{m+1} + 9 \times 10^m + (x_m - 1)10^{m-1} + x_{m-1} \times 10^{m-2} \\
& + x_{m-2} \times 10^{m-3} + \dots + x_3 \times 10^2 + x_2 \times 10 + x_1 \\
\therefore [b_n \cdots b_1] + [b_1 \cdots b_n] &= 11 \times 10^{2m} - 10^{2m-1} + 9 \times 10^{2m-2} + 9 \times \\
& 10^{2m-3} + \dots + 9 \times 10^{m+1} + 9 \times 10^m - 10^{m-1} \\
& = 11 \times 10^{2m} - 11 \times 10^{m-1} = 11(10^{2m} - 10^{m-1}) \\
& = 11(10^{n-1} - 10^{\frac{n-3}{2}})
\end{aligned}$$

2.  $a_{n-k+1} = a_k$

$$\text{此時 } [b_n \cdots b_1] = (10^{n-1} - 1)(a_n - a_1)$$

$$\text{令 } a_n - a_1 = k, k \in A, \text{ 則 } [b_n \cdots b_1] = (10^{n-1} - 1)k$$

仿上可推知

$$[b_1 \cdots b_n] = (10^{n-1} - 1)(11 - k)$$

$$\begin{aligned}
\therefore [b_n \cdots b_1] + [b_1 \cdots b_n] &= (10^{n-1} - 1)k + (10^{n-1} - 1)(11 - k) \\
& = 11 \times 10^{n-1} - 11 = 11(10^{n-1} - 1)
\end{aligned}$$

3.  $a_{n-k+1} < a_k, k = 2, 3, 4 \dots$  時

$$\begin{aligned}
[b_n \cdots b_1] &= (10^{n-1} - 1)x_1 - (10^{n-2} - 10)x_2 - (10^{n-3} - 10^2)x_3 \\
& - \dots - (10^m - 10^{m-1})x_m
\end{aligned}$$

ㄉ當  $n$  為偶數時，令  $n = 2m$

$$\begin{aligned}
[b_n \cdots b_1] &= (10^{2m-1} - 1)x_1 - (10^{2m-2} - 10)x_2 - (10^{2m-3} - 10^2)x_3 \\
& - \dots - (10^m - 10^{m-1})x_m
\end{aligned}$$

$$\begin{aligned}
\Rightarrow [b_1 \cdots b_n] &= 10^{2m} - x_1 \times 10^{2m-1} + (x_2 - 1)10^{2m-2} + x_3 \times 10^{2m-3} + \\
& x_4 \times 10^{2m-4} + \dots + x_m \times 10^m + (10 - x_m)10^{m-1} + \\
& (9 - x_{m-1})10^{m-2} + (9 - x_{m-2})10^{m-3} + \dots + (9 - x_2) \\
& \times 10 + (x_1 - 1)
\end{aligned}$$

$$\begin{aligned}
\therefore [b_n \cdots b_1] + [b_1 \cdots b_n] &= 10^{2m} - 10^{2m-2} + 10^m + 9 \times 10^{m-2} + 9 \\
& \times 10^{m-3} + \dots + 9 \times 10 - 1 = 10^{2m} - 10^{2m-2} + 10^m + \\
& 10^{m-1} - 10 - 1 = 10^n - 10^{n-2} + 10^{\frac{n}{2}} + 10^{\frac{n-1}{2}} - 11
\end{aligned}$$

ㄉ當  $n$  為奇數時，令  $n = 2m+1$

$$\begin{aligned}
[b_n \cdots b_1] &= (10^{2m} - 1)x_1 - (10^{2m-1} - 10)x_2 - (10^{2m-3} - 10^2)x_3 \\
& - \dots - (10^{m+1} - 10^{m-1})x_m
\end{aligned}$$

$$\Rightarrow [b_1 \cdots b_n] = 10^{2m+1} - x_1 \times 10^{2m} + (x_2 - 1) \times 10^{2m-1} + x_3 \times 10^{2m-2}$$

$$\begin{aligned}
& + x_4 \times 10^{2m-3} + \cdots + x_{m-1} \times 10^{m+2} + 0 \times 10^{m+1} + x_m \times \\
& 10^m + (10 - x_m) 10^{m-1} + (9 - x_{m-1}) 10^{m-2} + (9 - x_{m-2}) \\
& 10^{m-3} + \cdots + (9 - x_2) \times 10 + (x_1 - 1) \\
\therefore [b_n \cdots b_1] + [b_1 \cdots b_n] &= 10^{2m+1} - 10^{2m-1} + 10^m + 9 \times 10^{m-2} + \\
& 9 \times 10^{m-3} + \cdots + 9 \times 10^2 + 9 \times 10 - 1 \\
& = 10^{2m+1} - 10^{2m-1} + 10^m + 10^{m-1} - 10 - 1 \\
& = 10^n - 10^{n-2} + 10^{\frac{n-1}{2}} + 10^{\frac{n-3}{2}} - 11
\end{aligned}$$

(七) 討論

1. 若  $n$  位數中，  $a_{n-k+1} = a_k$ ，  $k = 1, 2, 3 \cdots m$  ( $n = 2m \vee 2m+1$  時)

$$[b_n \cdots b_1] = 0 \quad \text{故 } [b_1 \cdots b_n] = 0$$

$$\therefore [b_n \cdots b_1] + [b_1 \cdots b_n] = 0$$

如二位數，三位數其值均為零。

2. 若  $a_{n-k+1}$  與  $a_k$  中  $a_n = a_1$

$$\begin{aligned}
\text{則 } [b_n \cdots b_1] &= (10^{n-2} - 10)(a_{n-1} - a_2) + \cdots + (10^{n-k} - 10^{k-1}) \\
&\quad (a_{n-k+1} - a_k)
\end{aligned}$$

仿上討論亦可得  $[b_n \cdots b_1] + [b_1 \cdots b_n]$  仍為一定值

### 三、結論

(一) 任意寫出一個  $n$  位數，(用分類歸納法，在某種限制條件下，所得到具有相同形式的數)，求該數減去該數倒寫所成之  $n$  位數之差再加上此差倒寫所成之數，必會等於某定值。

(二) 由  $[b_n b_{n-1} \cdots b_2 b_1] = (10^{n-1} - 1)(a_n - a_1) + (10^{n-2} - 10)(a_{n-1} - a_2) + \cdots + (10^{n-k} - 10^{k-1})(a_{n-k+1} - a_k)$

在  $a_n > a_1$  下，由  $a_{n-k+1}$  與  $a_k$  之大小可推知

$[b_n b_{n-1} \cdots b_2 b_1] + [b_1 b_2 \cdots b_{n-1} b_n]$  會等於某一種定值

且當  $n$  為偶數時，其定值至多有  $3^{\frac{n-2}{2}}$  個， $n \geq 2$

當  $n$  為奇數時，其定值至多有  $3^{\frac{n-3}{2}}$  個， $n \geq 3$

如 1. 二位數，定值為 99

2. 三位數，定值為 1089

3. 四位數，定值 10890, 10989, 9999 等 3 個
4. 五位數，定值為 109890, 109989, 99099 等 3 個
5. 六位數，定值為 109890, 1099890, 1089990, 991089,  
1099989, 1090089, 99099, 99999 等 8 個
6. 七位數，定值為 10998900, 10999890, 10890990, 9900099  
9901089, 10008999, 10999989, 10891089 等 8 個

(三)由  $a_{n-k+1}$  與  $a_k$  之大小可推斷出，定值必成一特殊的規則。

如  $11(10^{n-1} - 10^{\frac{n-2}{2}})$ ,  $11(10^{n-1} - 10^{\frac{n-3}{2}})$ ,  $11(10^{n-1} - 1)$ ,  
 $10^n - 10^{n-2} + 10^{\frac{n}{2}} + 10^{\frac{n-2}{2}} - 11 \dots$  等

(四)由  $[b_n, b_{n-1} \dots b_2, b_1]$  推論出其倒寫  $[b_1, b_2, \dots b_{n-1}, b_n]$  而得一個一般公式是一大收穫。

## 評語

1. 對於數字的規律具有敏銳的觀察力，並能將各種出現的情形，加以歸納成為一般性的結論。
2. 善用數學的基本知識推演出新的結果。對於國中生而言，誠屬難能可貴。